Modelling of Premixed Flame under Harmonic Oscillation

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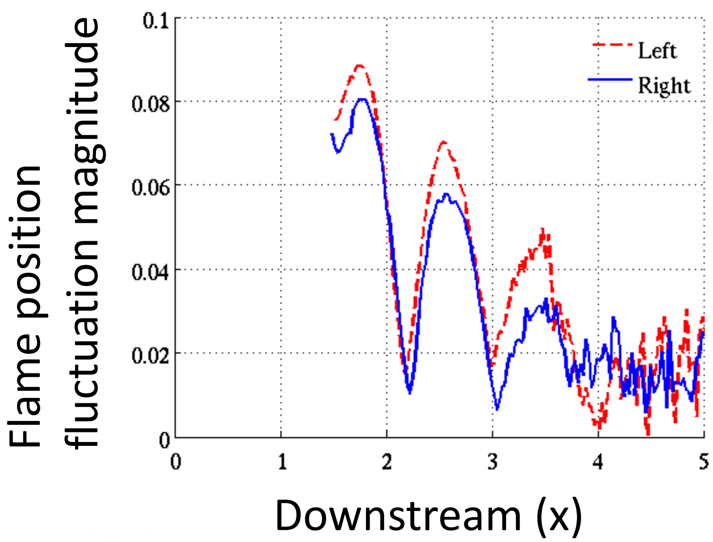
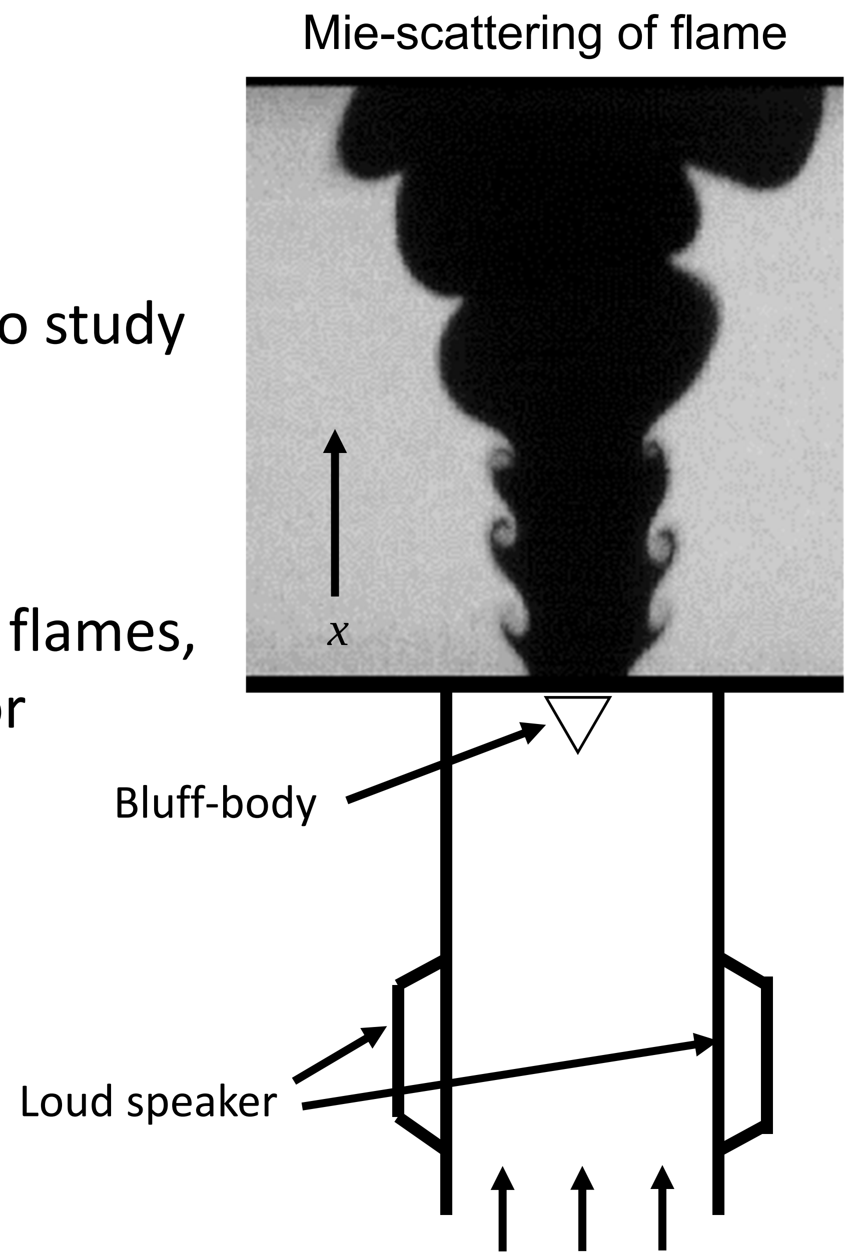
Introducing
Flame-Holder Oscillation

Conventional ways to force flames

- Conventionally, we force the flow to study combustion instability
  - Developed flame wrinkle is not sinusoidal
  - Flow field continues to perturb flames, leading to interference behavior





Bluff-body flame, velocity-foced, courtesy of Shanbhogue

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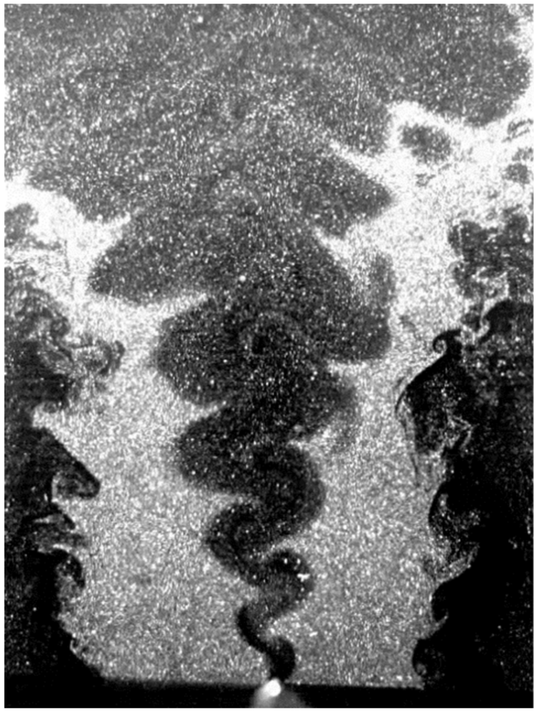
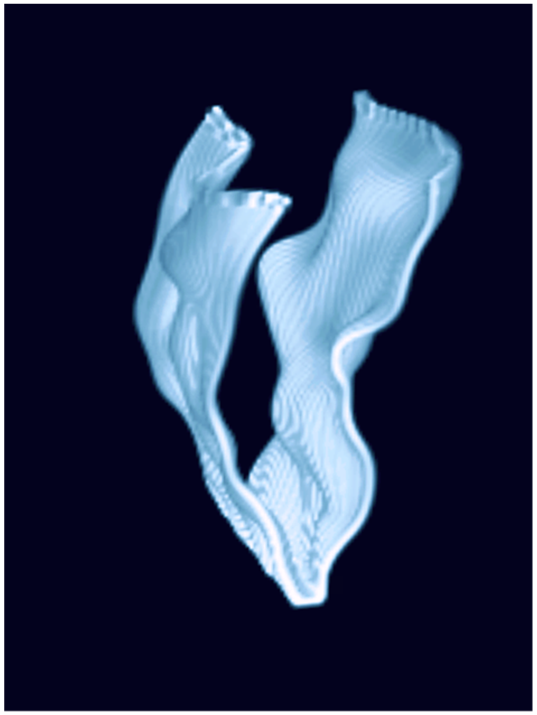
Flame-holder oscillation

- Instead, we vibrate the flame holder
  - Flow field is nominally uniform
  - The flame wrinkle starts from a sine wave shape

Experimental setup
of Humphrey et al.



Mie-scattering of the flame



On-going DNS
By Shin, Aspden, Lieuwen

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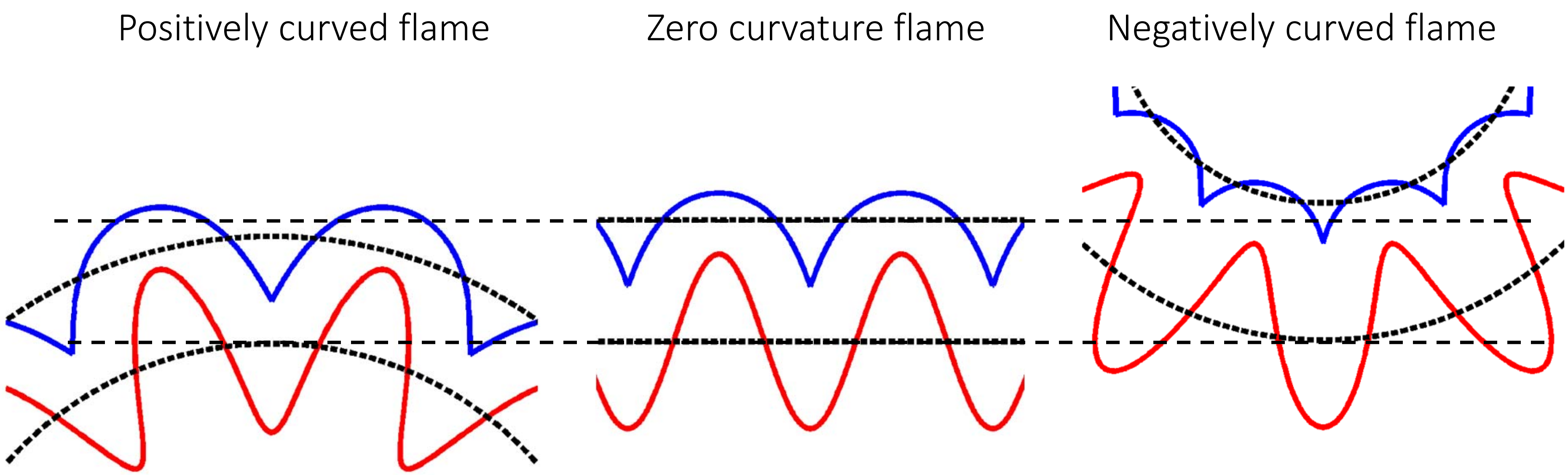
Turbulent Markstein Length

Turbulent Markstein Length: Curvature effect due to small scale wrinkles

- Consider a condition that laminar flame speed is constant, i.e., independent of curvature.
- Supposed 3 different initial flame positions

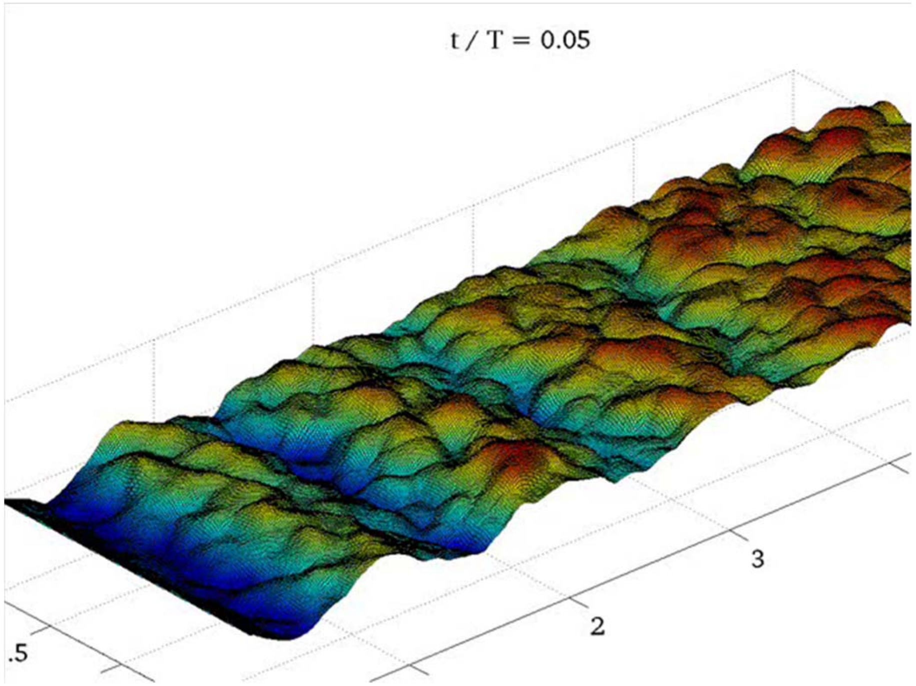


- Propagation speed, on average, depends on the curvature of ensemble averaged front

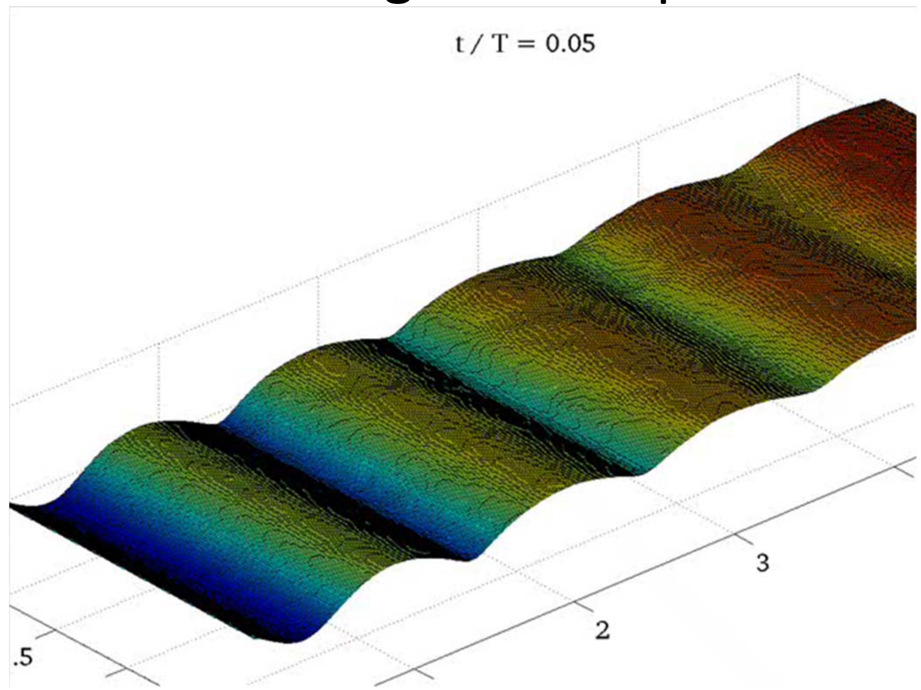
Turbulent Markstein length

- **Turbulent flame speed** depends on the **averaged curvature**

Instantaneous flame position



Phase-averaged flame position



By 3D G-equation simulation

$$u_{rms} / u_0 = 7\ \%$$

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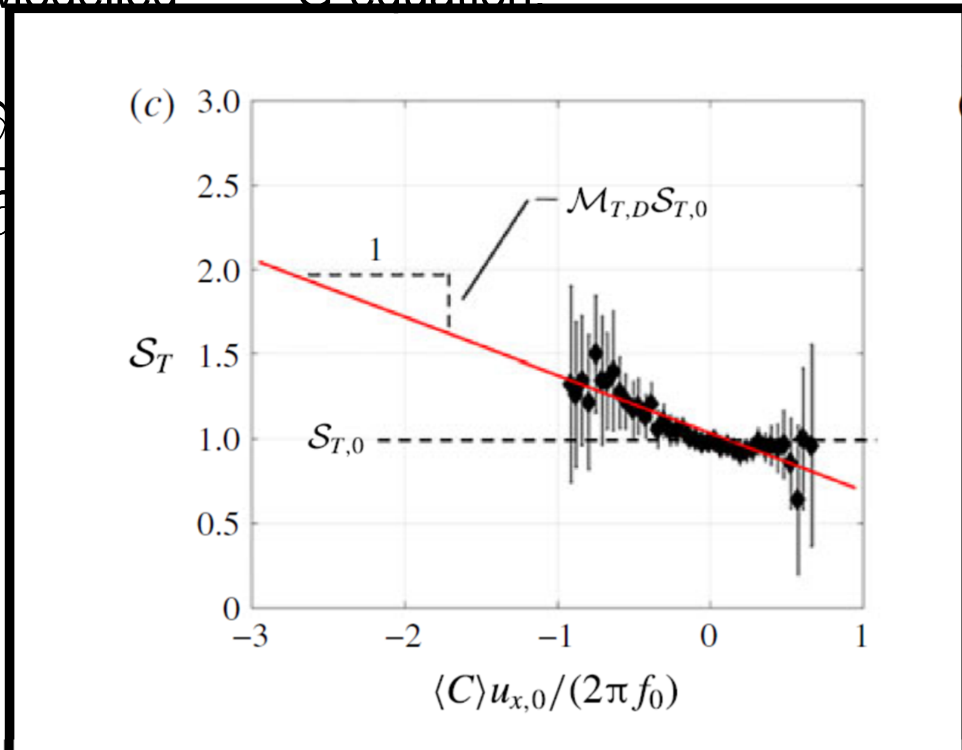
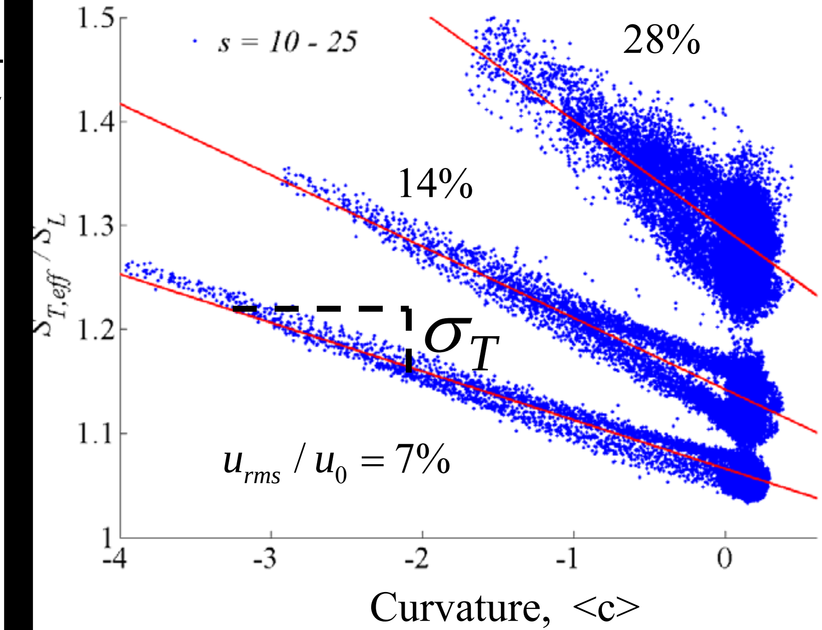
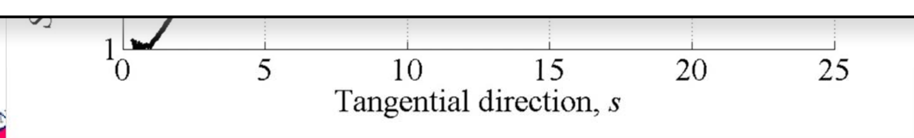
Turbulent Flame Speed







Nearly linear correlation with flame

…kstein Length," σ_T

$$S_{T,eff} = S_{T,eff}\left(1-\sigma_T\left\langle c\right\rangle\right)$$

This dependency is verified experimentally recently (Humphrey, Emerson, Lieuwen, JFM, 2018)

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Quantifying
the Curvature effect

High order asymptotic approach

Example: $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

- First order asymptotic method:

If we decompose the velocity into: $u = u_0 + u'$

$$\frac{\partial u'}{\partial t} + u_0\frac{\partial u'}{\partial x} = 0$$

Which has an solution: $u' = u'_{init}\left(x - \frac{t}{u_0}\right)$

- Second order asymptotic method:

Expand in the orders of fluctuation magnitudes: $u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2$

$$\varepsilon\left[\frac{\partial u_1}{\partial t} + u_0\frac{\partial u_1}{\partial x}\right] + \varepsilon^2\left[\frac{\partial u_2}{\partial t} + u_1\frac{\partial u_1}{\partial x} + u_0\frac{\partial u_2}{\partial x}\right] + O\left(\varepsilon^3\right) = 0$$

O(ε): $$\frac{\partial u_1}{\partial t} + u_0\frac{\partial u_1}{\partial x} = 0$$

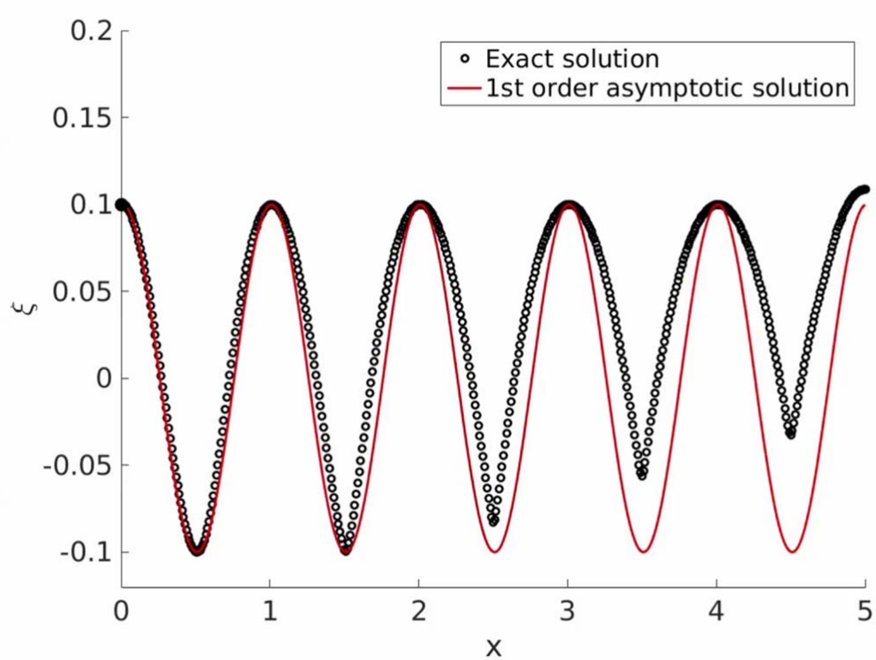
Get the solution for u_1.

O(ε²): $$\frac{\partial u_2}{\partial t} + u_0\frac{\partial u_2}{\partial x} = -u_1\frac{\partial u_1}{\partial x}$$

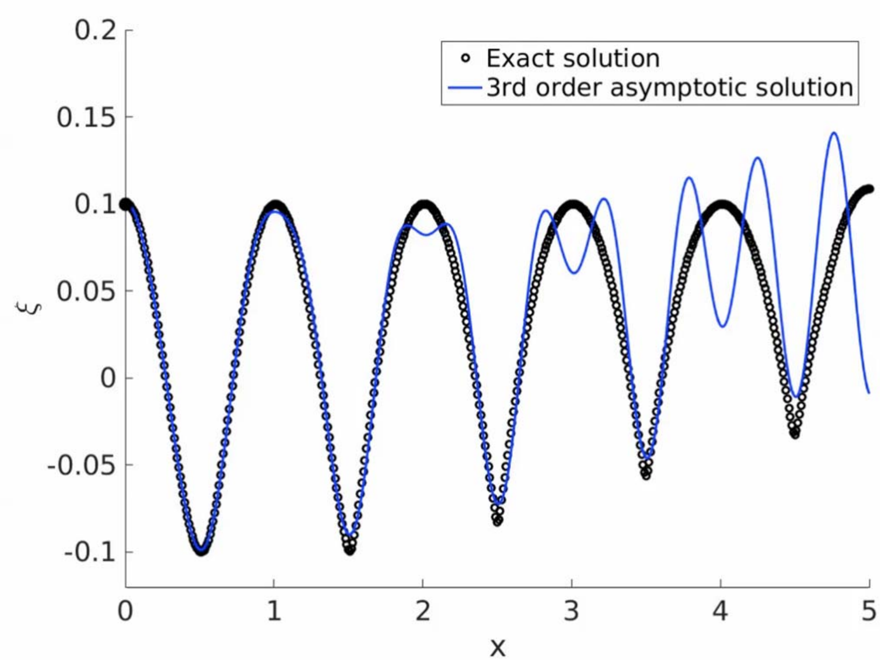
Get the solution for u_2.

Comparison of exact vs asymptotic solutions

1st order asymptotic solution

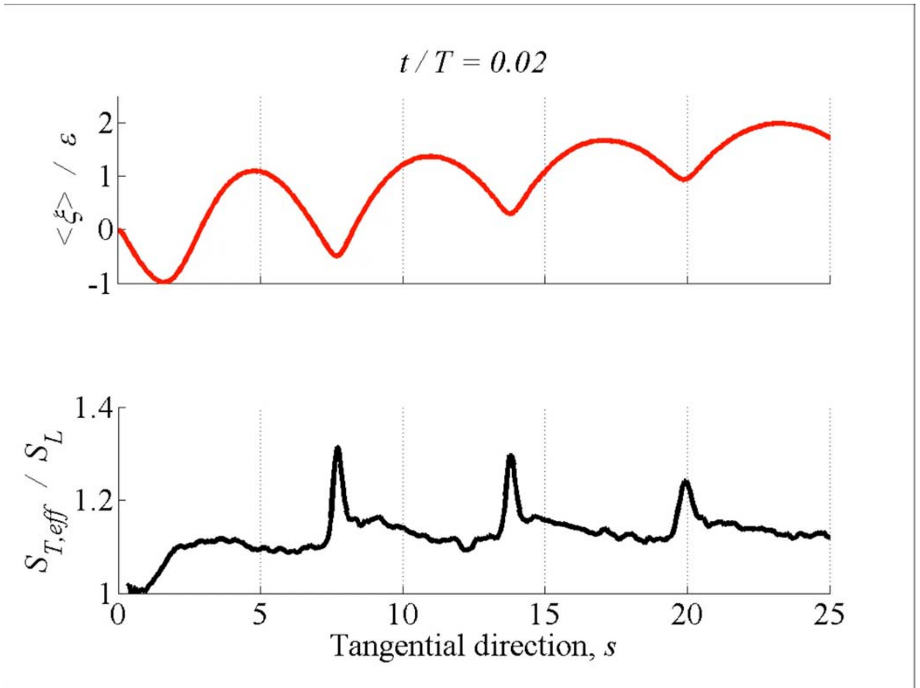


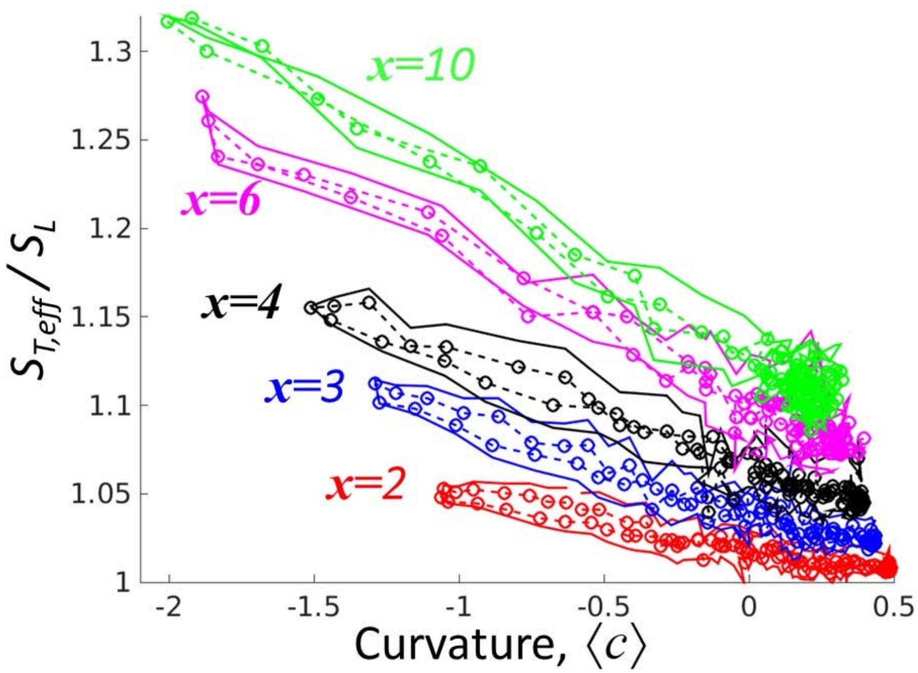
3rd order asymptotic solution



Higher order asymptotic solution works better near x=0, but deviates more further downstream.

Turbulent Markstein length at different locations



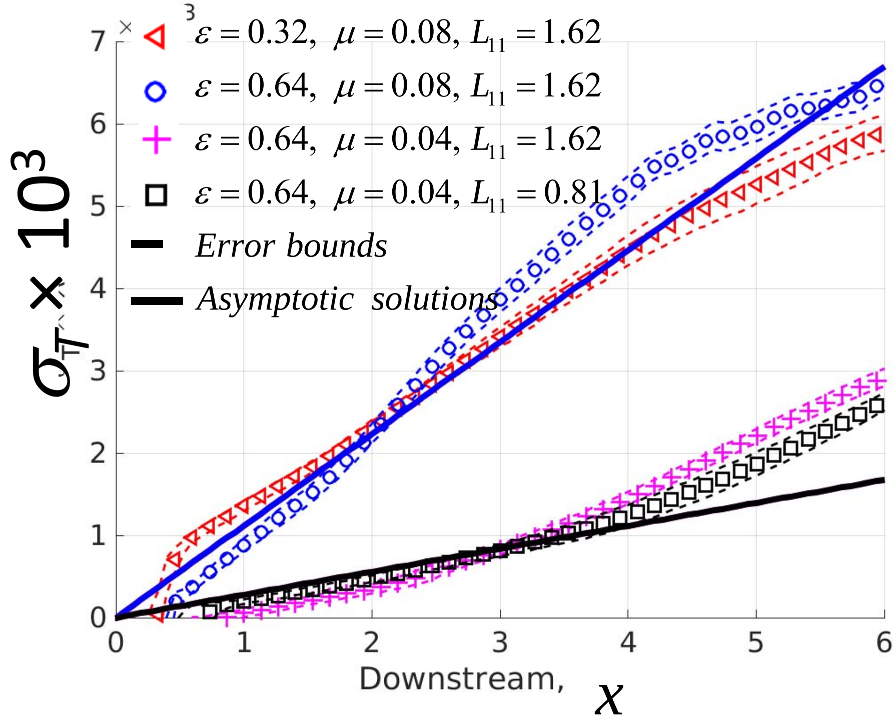


Turbulent Markstein Length for Iso-thermal condition

- Simulations done by solving G-equation with frozen flow
- Asymptotic expansion up to the 3rd order

$$\sigma_T \simeq \mu^2 \left(\frac{S_L}{u_0} + \frac{u_0}{S_L} \right) x$$

μ: turbulent intensity



$$\text{where}\quad S_{T,eff} = S_{T,0}\left(1 - \sigma_T \langle C \rangle\right)$$

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Far field asymptotic solution for Turbulent Markstein length

$$\sigma_T = \left[\left\langle \frac{\partial \xi'}{\partial t} \frac{\partial v'}{\partial y} \right\rangle - \frac{1}{2} \frac{\partial \left\langle v'^2 \right\rangle}{\partial y} \right] - \frac{1}{S_L} \left[\left\langle \xi' \frac{\partial^2 v'}{\partial y^2} \right\rangle + \left\langle \frac{\partial \xi'}{\partial t} \frac{\partial u'}{\partial y} \right\rangle - \left\langle \frac{\partial \xi'}{\partial z} \frac{\partial w'}{\partial y} \right\rangle - \frac{\partial \left\langle u' v' \right\rangle}{\partial y} \right]$$

u', *v'*, *w'*: fluctuating velocity components

ξ': fluctuating flame position

- This has not been validated with simulations yet.

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Conclusion

- Oscillating flame holder setup is an interesting case to study unsteady flame dynamics
- Turbulent flame speed can depends on curvature
- Asymptotic analysis indicates that the turbulent Markstein length is proportional to squared turbulent intensity and laminar flame speed

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