Evaluations of turbulent burning velocity and wall heat flux using integral energy equation for premixed flame-wall interaction in turbulent boundary layers

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Outline

- Introduction
- Motivation and Objectives
- DNS code and numerical setup
- Validation with previous DNS data of non reacting channel flow
- Energy Integral Equation
- Results
- Summary

Introduction

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- Flame-wall interaction (FWI) occurs in many flows of engineering interest (e.g., Spark Ignition (SI) engines and gas turbines), and modelling of these events remains challenging.
- The turbulence structure is altered by the walls, and the interaction of flame elements with walls leads to modifications of the underlying combustion process.
- Spatial and temporal fluctuations of wall temperature induce thermal stresses and strongly affect combustor lifetimes.
- FWI is increasingly becoming important as new combustors are being made smaller to increase energy density and reduce weight.
- Without near-wall treatment, the well-known reaction rate closure models do not correctly reflect the influence of the wall and also reaction rate closures in RANS and LES do not account for boundary layer in the modelling approach.

Motivation & Objectives

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- ❖ Experimentally mean velocity, mean temperature and wall heat flux within the TBLs can be measured with required level of confidence.
- ❖ During experiments turbulent burning velocity is very difficult to measure within TBLs

- ➢ An integral form of the energy conservation equation has been derived from the first principle for low Mach number conditions under a quasi-stationary state
- ➢ To establish the relation between turbulent burning velocity and wall heat flux
- ➢ To estimate the turbulent burning velocity within TBLs with the measurements of mean velocity, temperature and wall heat flux.

Direct Numerical Simulation

- A well-known three-dimensional compressible DNS code SENGA+[1] is used.
- The code solves conservation equations for mass, momentum, energy and chemical species in non-dimensional form using finite difference method.
- The spatial derivatives are evaluated via a 10th order for internal points and gradually decreasing to 2nd order at the non-periodic boundaries.
- A Runge-Kutta (3rd order explicit) scheme for time advancement.
- Single step chemistry representing the stoichiometric methane-air flame is considered for the computational economy.
- A DNS database of oblique wall quenching (OWQ) for a V-flame in a turbulent channel flow is considered [2].
- This configuration is similar to previous DNS studies by Alshaalan & Rutland[3] and Gruber et al.[4].
- The wall temperature is set to that of the non-reacting air-fuel mixture.
- The flame is representative of methane $\phi = 1.0$, $Le = 1.0$ with $S_L/u_\tau = 0.7$.

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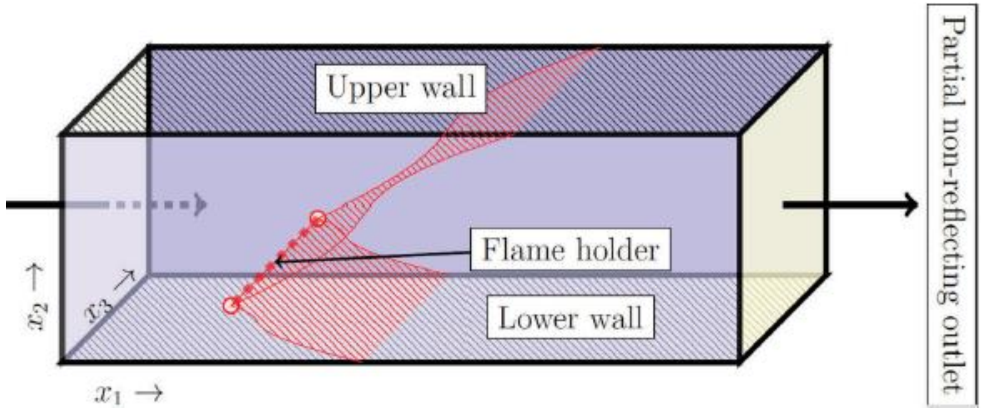
[1] **K. Jenkins, R.S. Cant** *Recent Advances in DNS and LES: Proceedings of the 2nd AFOSR Conference (1999)*
[2] **U. Ahmed, N. Chakraborty, M. Klein** *Flow Turbul. Combust., 106 (2021), pp. 701-732*
[3] **T. M. Alshaalan, CJ Rutland** *Symposium (International) on Combustion 27* (1998)(1), 793-799
[4] **A. Gruber, R. Sankaran, E. R. Hawkes, J. H. Chen** *Journal of Fluid Mechanics* (2010), 658, 5-32

V-flame simulation

Turbulent channel inlet



$$\boldsymbol{\tau_{wall}} = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}, \quad \boldsymbol{u_\tau} = \sqrt{\frac{|\boldsymbol{\tau_{wall}}|}{\boldsymbol{\rho}}}, \quad \boldsymbol{Re_\tau} = \frac{\boldsymbol{\rho u_\tau h}}{\boldsymbol{\mu}} \text{ and } \boldsymbol{y}^+ = \frac{\boldsymbol{u_\tau y}}{\boldsymbol{\nu_{wall}}}$$

- V-flame is investigated in the $Re_\tau = 110$ channel flow with inert walls [1].
- The flame holder is placed in the log-layer region of the channel flow at $y^+ = 55$.
- Domain size $10.69h \times 2h \times 4h$ discretised on $1920 \times 360 \times 720$ (approx. 0.5 billion) grid points.
- The simulation is run for three flow through times after the initial transients have decayed.
- Progress variable is defined in terms of the fuel mass fraction.

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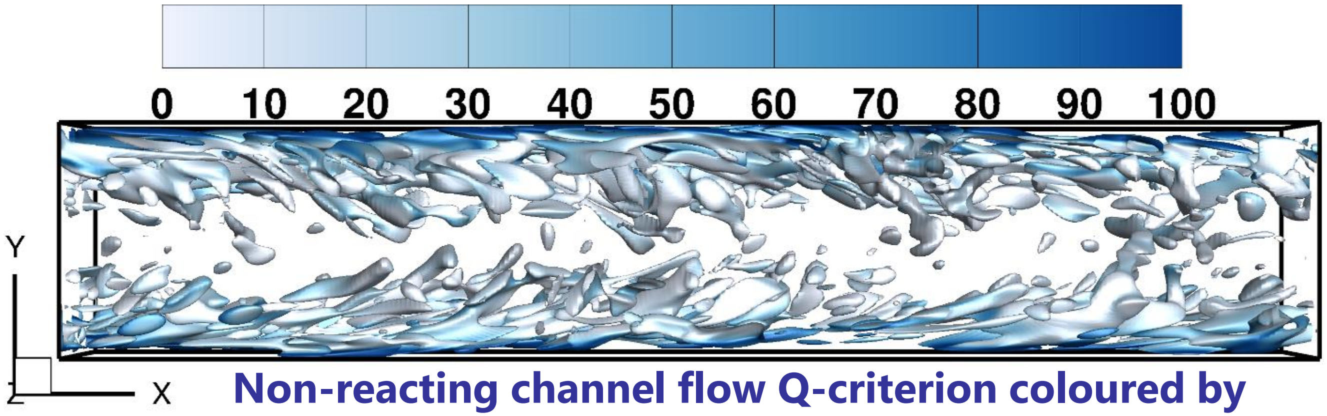
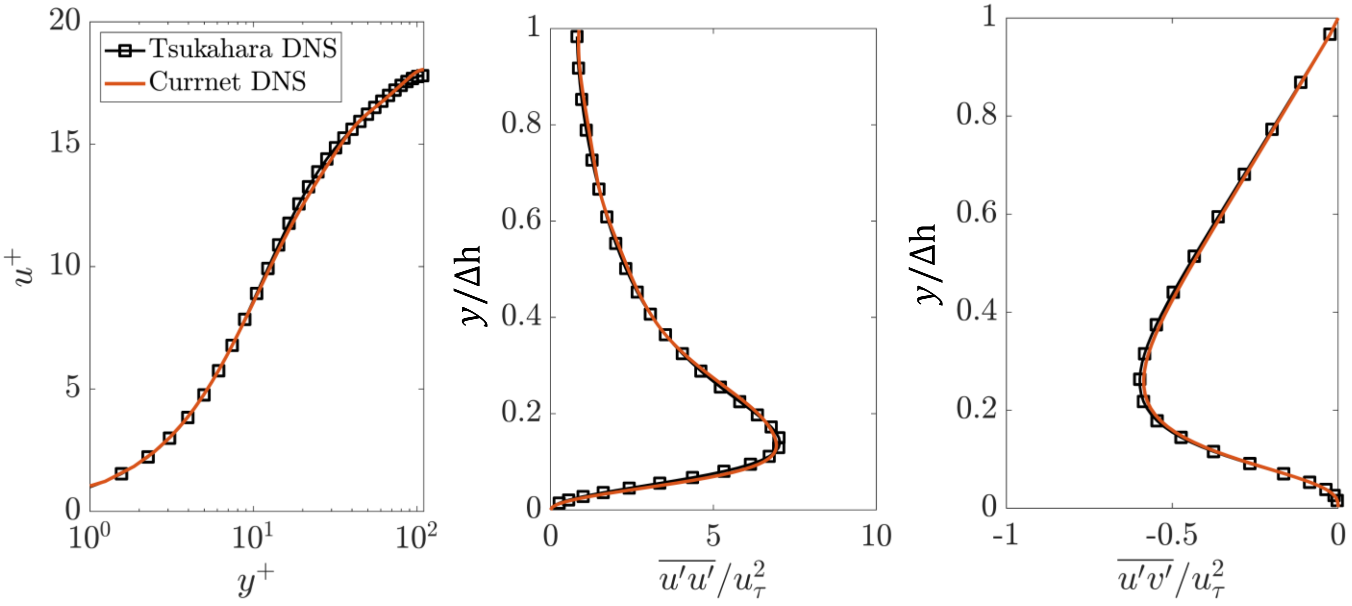
[1] **U. Ahmed, N. Chakraborty, M. Klein** *Flow Turbul. Combust., 106 (2021), pp. 701-732*

Non-reacting flow simulation



Non-reacting channel flow Q-criterion coloured by vorticity magnitude



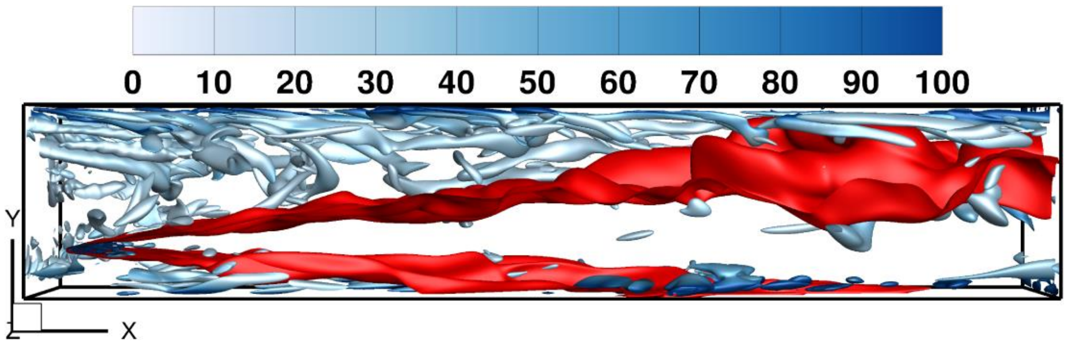
Non-reacting channel flow mean velocity and Reynolds stress profiles

- ❖ $Re_\tau = 110$ for the non-reacting channel.
- ❖ $y^+ = 0.6$ at the wall approximately two grid points are in $y^+ = 1.0$ region.
- ❖ Domain size $10.69h \times 2h \times 4h$.
- ❖ $1920 \times 360 \times 720$ (approx. 0.5 billion) grid points.
- ❖ Data validated against existing DNS.
- ❖ Data is recorded in time on the $(y - z)$ plane.

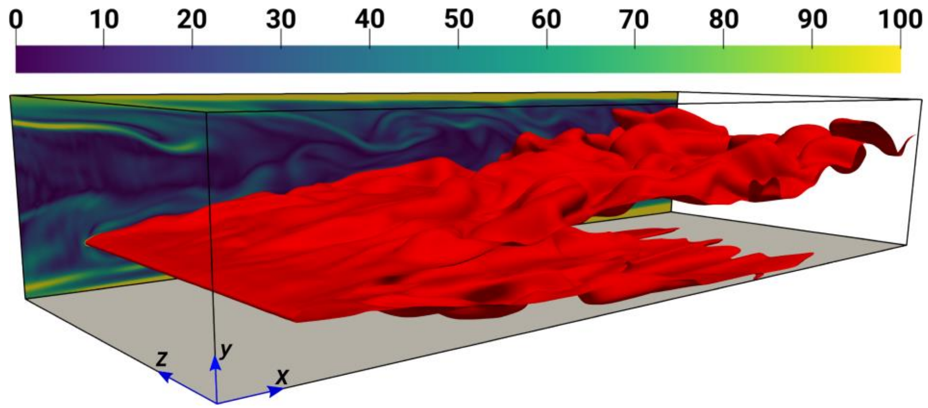
DNS data of Tsukahara et al. is available at **http://www.rs.tus.ac.jp/~t2lab/db/index.html**

V-flame instantaneous behaviour

V-flame with isothermal walls

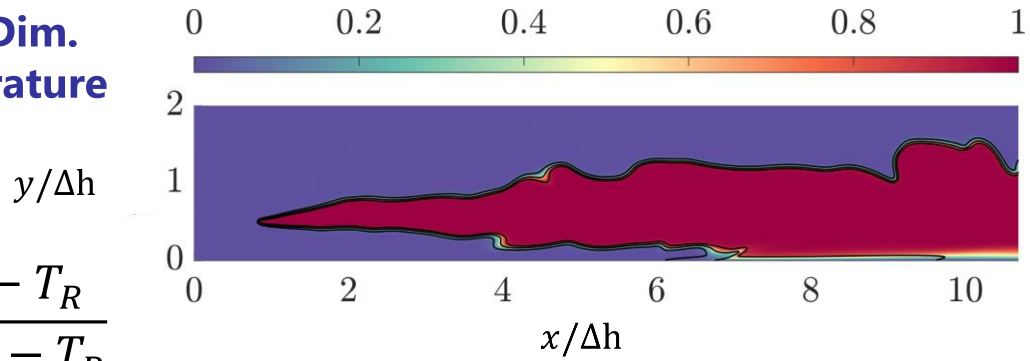


Q-criterion coloured by vorticity magnitude



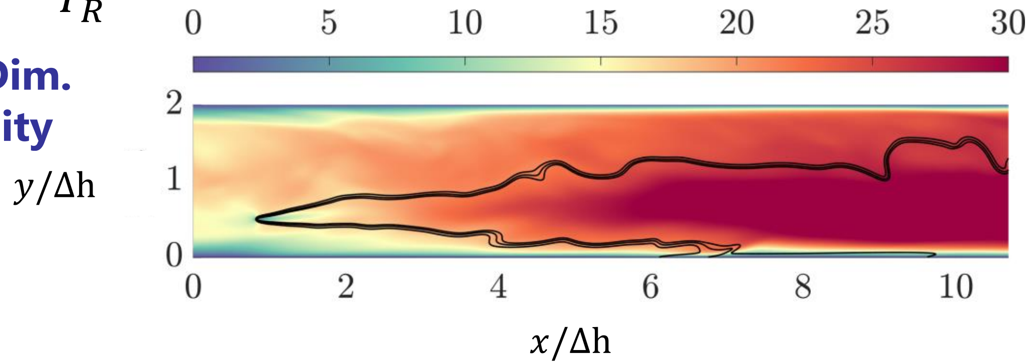
Vorticity magnitude

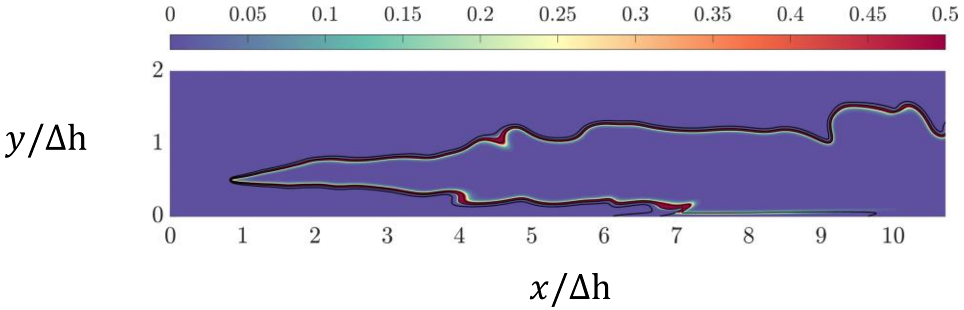
Non-Dim. Temperature



$$\theta = \frac{\hat{T} - T_R}{T_{ad} - T_R}$$

Non-Dim. Velocity





Non-dimensional reaction rate

Energy integral equation: Premixed flame-wall interaction in turbulent boundary layers

Reynolds averaged Energy eq. for a flat plate boundary layer with small values of Mach number for a statistically steady-state

$$\frac{\partial(\bar{\rho}\tilde{u}\tilde{h})}{\partial x} + \frac{\partial(\bar{\rho}\tilde{v}\tilde{h})}{\partial y} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} + \overline{\dot{\omega}_T}$$

After integration from the wall to the thermal boundary layer thickness δ_T (i.e., the wall normal distance where $\left(\partial\tilde{\theta}/\partial y\right)_{y=\delta_T} = 0$)

$$\frac{\partial}{\partial x}\int_0^{\delta_T} \bar{\rho}\tilde{u}(\tilde{h} - \tilde{h}_\infty)dy + \frac{d\tilde{h}_\infty}{dx}\int_0^{\delta_T} \bar{\rho}\tilde{u}\,dy = \bar{q}_w + \int_0^{\delta_T} \overline{\dot{\omega}_T}dy$$

Energy integral equation: Premixed flame-wall interaction in turbulent boundary layers

$$\underbrace{\frac{\partial}{\partial x}\int_0^{\delta_T}\frac{\bar{\rho}\tilde{u}\left(\tilde{h}-\tilde{h}_\infty\right)}{\rho_0 u_{\tau,NR}(h_{ad}-h_0)}dy}_{T_1}+\underbrace{\frac{1}{(h_{ad}-h_0)}\frac{d\tilde{h}_\infty}{dx}\int_0^{\delta_T}\frac{\bar{\rho}\tilde{u}}{\rho_0 u_{\tau,NR}}dy}_{T_2}=\underbrace{\frac{\bar{q}_w}{\rho_0 u_{\tau,NR}(h_{ad}-h_0)}}_{T_3}+\underbrace{\frac{1}{\rho_0 u_{\tau,NR}}\int_0^{\delta_T}\overline{\dot{\omega}_c}dy}_{T_4}$$

Advection Term

Freestream temperature variation term

Wall heat flux term

Mean reaction rate term

After integration in the streamwise direction from x=L_1 to x=L_2

$$T_{1L}+T_{2L}=-Nu_L/\{Re_\tau Pr\}+A_{proj}S_T/[A_{seg}u_{\tau,NR}]$$

Energy integral equation: Premixed flame-wall interaction in turbulent boundary layers

$$T_{1L} + T_{2L} = -Nu_L/\{Re_\tau Pr\} + A_{proj} S_T/[A_{seg} u_{\tau,NR}]$$

$$T_{1L} = (L_2 - L_1)^{-1} \int_{L_1}^{L_2} T_1 dx \ ,$$

$$T_{2L} = (L_2 - L_1)^{-1} \int_{L_1}^{L_2} T_2 dx \ ,$$

$Nu_L = (L_2 - L_1)^{-1} \int_{L_1}^{L_2} Nu_x dx$ is the mean Nusselt number

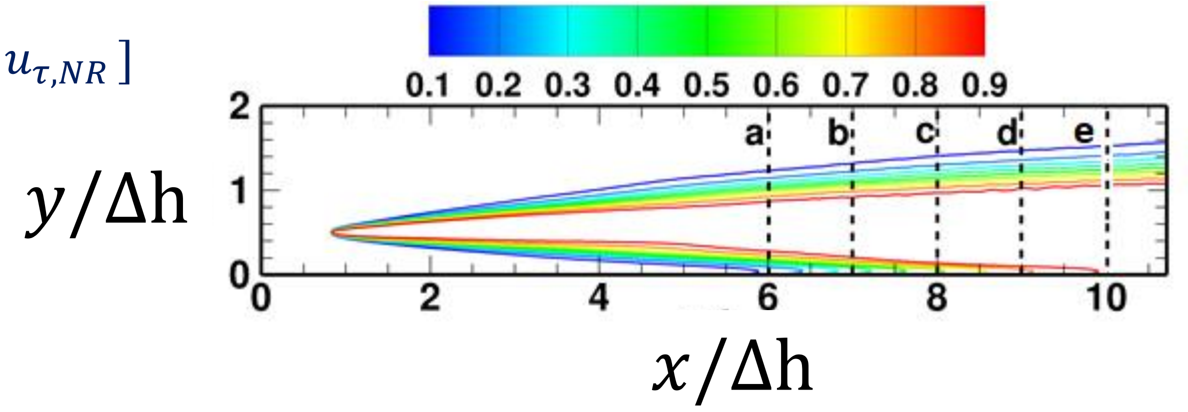
A_{proj} is the projected flame surface area

$A_{proj} = W(L_2 - L_1)/\cos\phi$ with $\phi = 3.07^0$ with $\tilde{c} = 0.94$ with the x −axis.

The iso-contours of smaller values of $\tilde{c}$ make an angle in the range of $\phi = 4^0 - 5^0$ with the x-axis.

$A_{seg} = (L_2 - L_1)W$ is the area of the segment

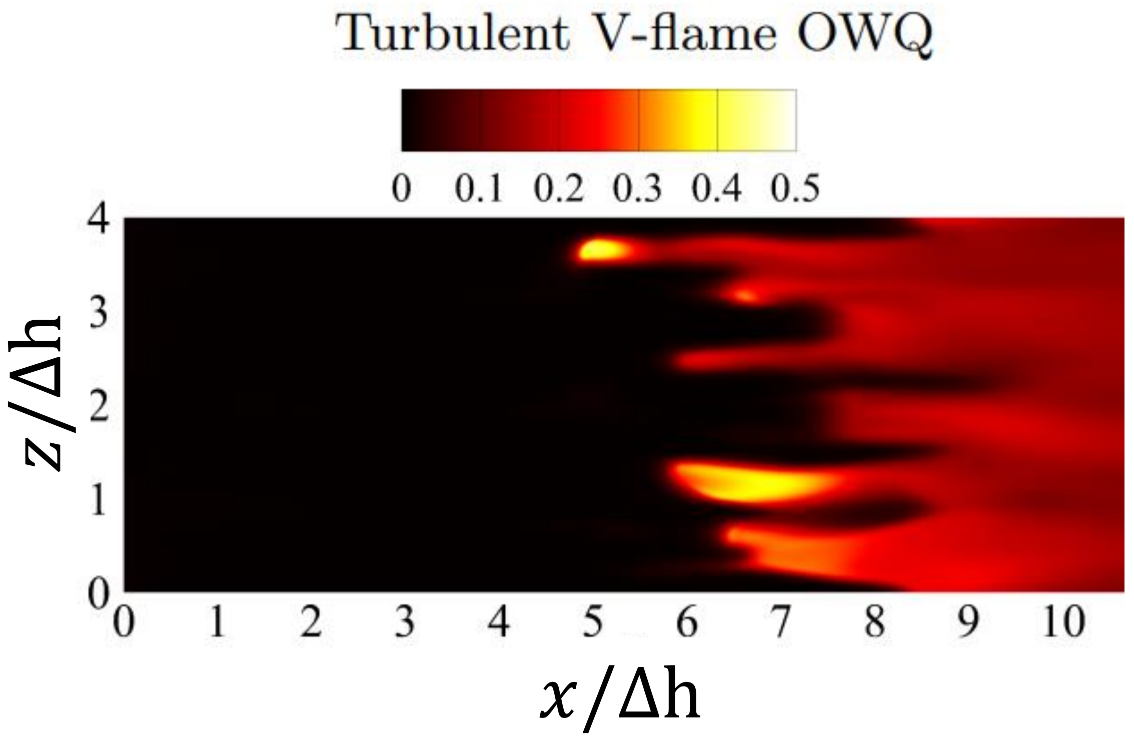
$S_T = \left(\rho_0 A_{proj}\right)^{-1} \int_{L_1}^{L_2} \int_0^{\delta_T} \overline{\dot{\omega}_c} W dx dy$ is the turbulent burning velocity

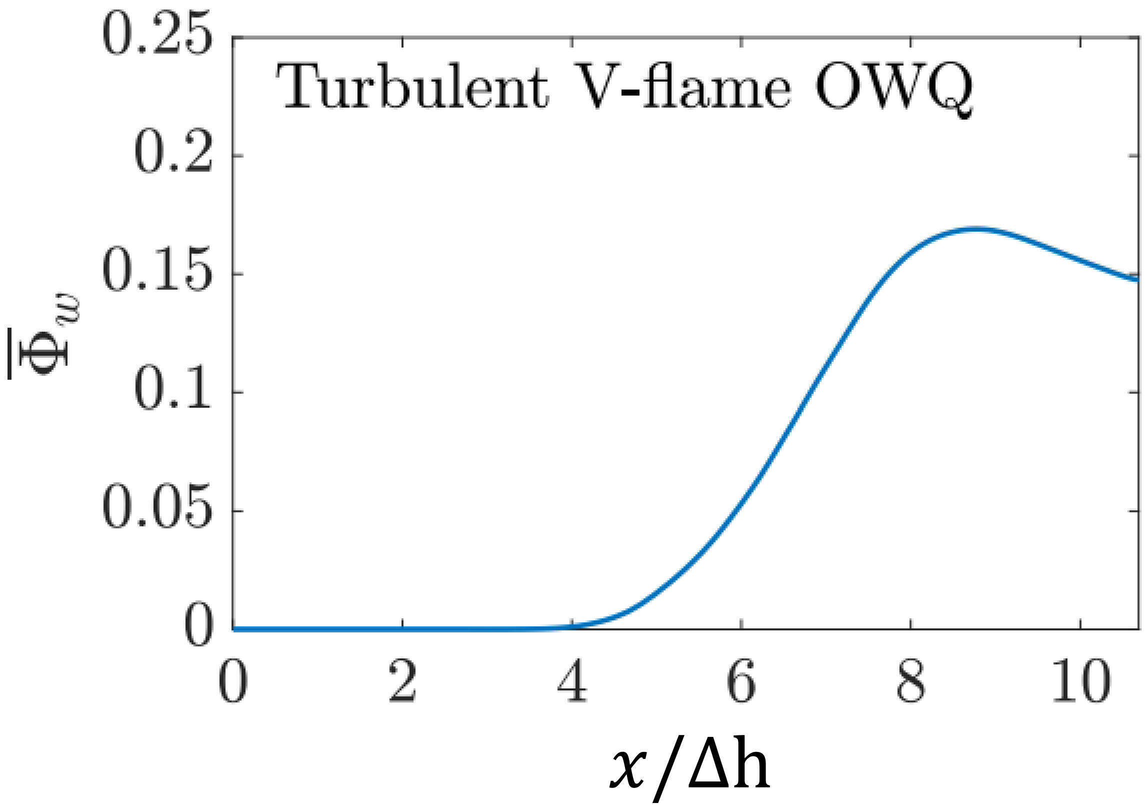


Favre Averaged Progress variable

- Location $a = x/\Delta h = 6$
- Location b $= x/\Delta h = 7$
- Location c $= x/\Delta h = 8$
- Location d $= x/\Delta h = 9$
- Location e $= x/\Delta h = 10$

Mean Wall Heat Flux

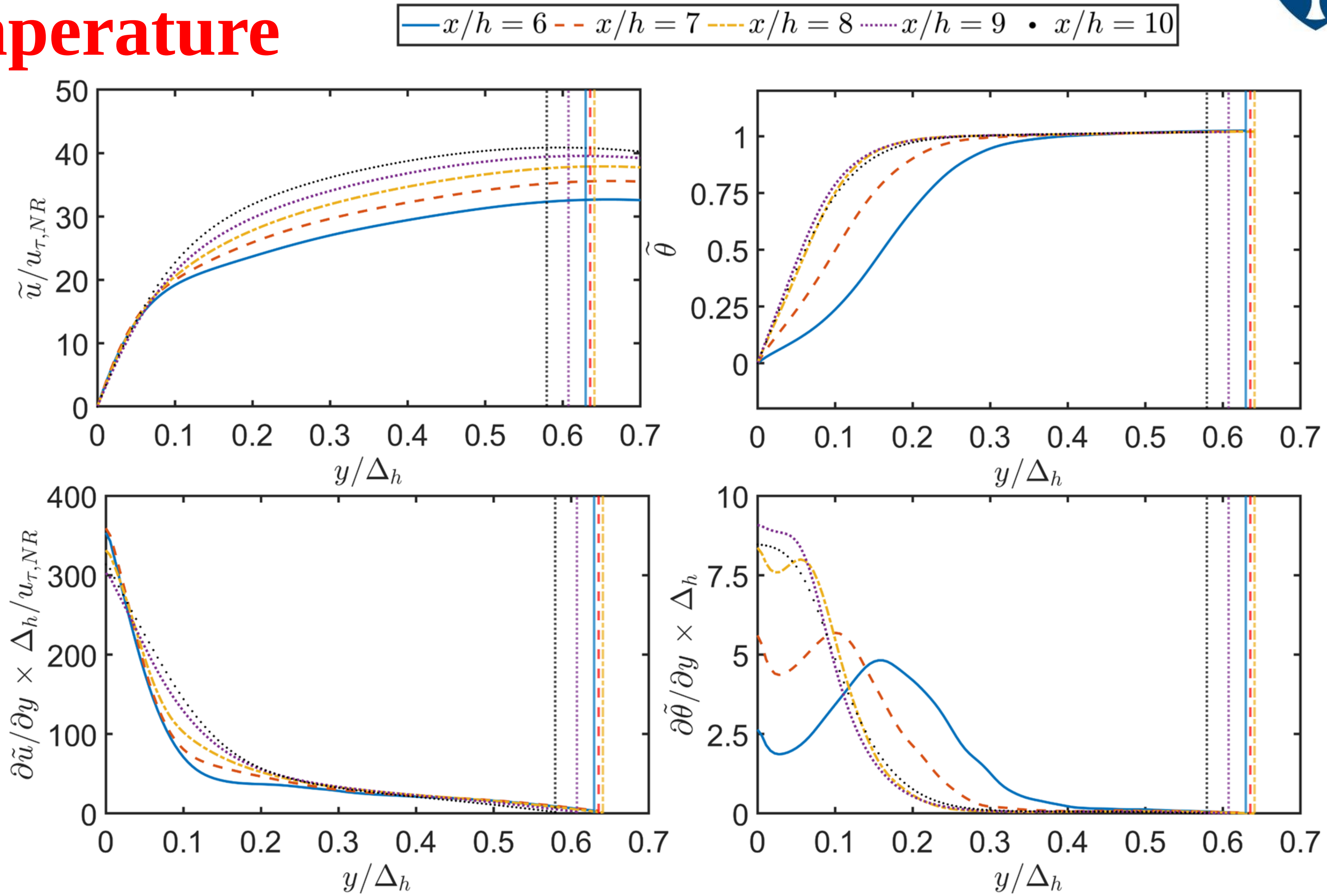




$$\overline{\Phi}_w = |\bar{q}_w|/[\rho_0 S_L C_p (T_{ad} - T_0)]$$

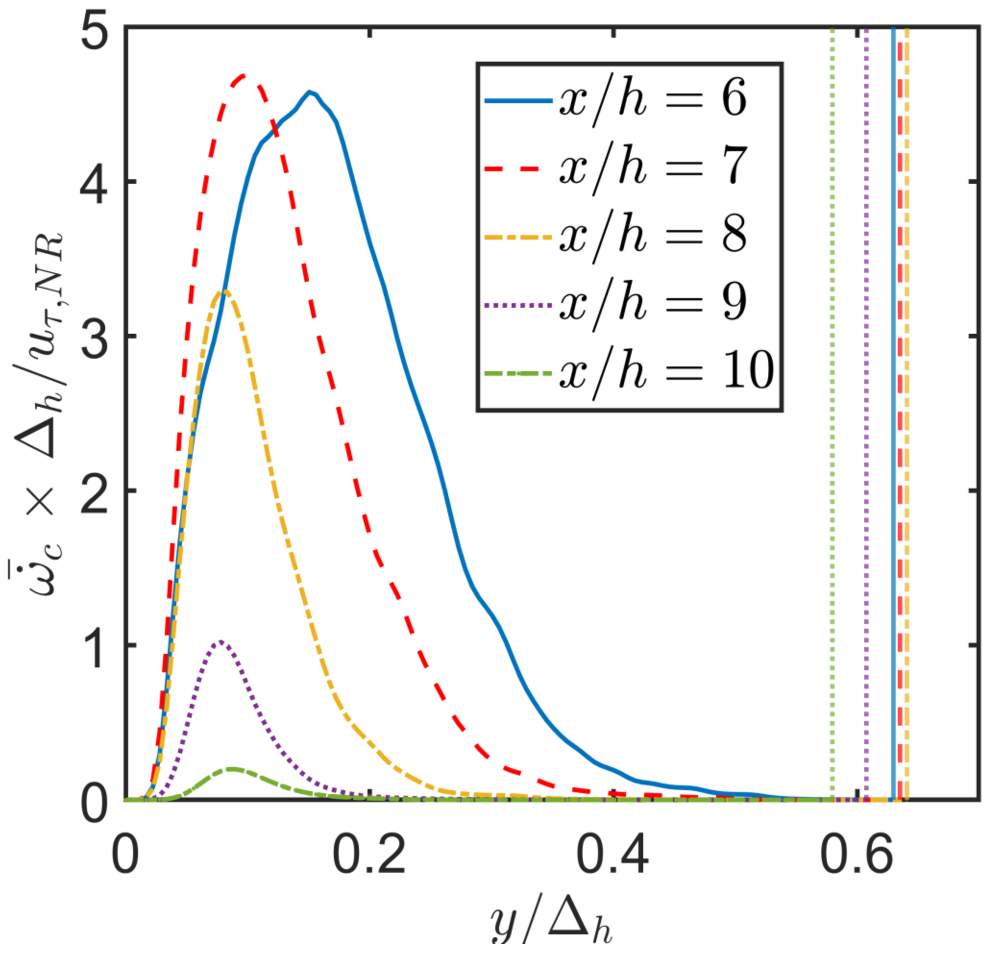
The behaviour of instantaneous (left) and mean (right) normalised wall heat flux, $\overline{\Phi_w}$, along the bottom wall for the V-flame OWQ configuration

Distribution of mean and gradient of velocity and temperature



Distributions of (a) $\tilde{u}/u_{\tau,NR}$, (b) $\tilde{\theta}$, (c) $\partial\tilde{u}/\partial y \times \Delta_h/u_{\tau,NR}$ and (d) $\partial\tilde{\theta}/\partial y \times \Delta_h$ with y/Δ_h for $x/\Delta_h = 6.0, 7.0, 8.0, 9.0$ and 10.0. Vertical lines represents the cut-off limits of the thermal boundary layer thickness δ_t.

Mean reaction rate



Variations of $\overline{\dot{\omega}_c} \times \Delta_h/\rho_0 u_{\tau,NR}$ (lines) with y/Δ_h for the bottom wall for x/Δ_h= 6.0, 7.0, 8.0, 9.0 and 10.0. Vertical lines represents the cut-off limits of the thermal boundary layer thickness δ_t.

Variation of terms in the wall normal direction

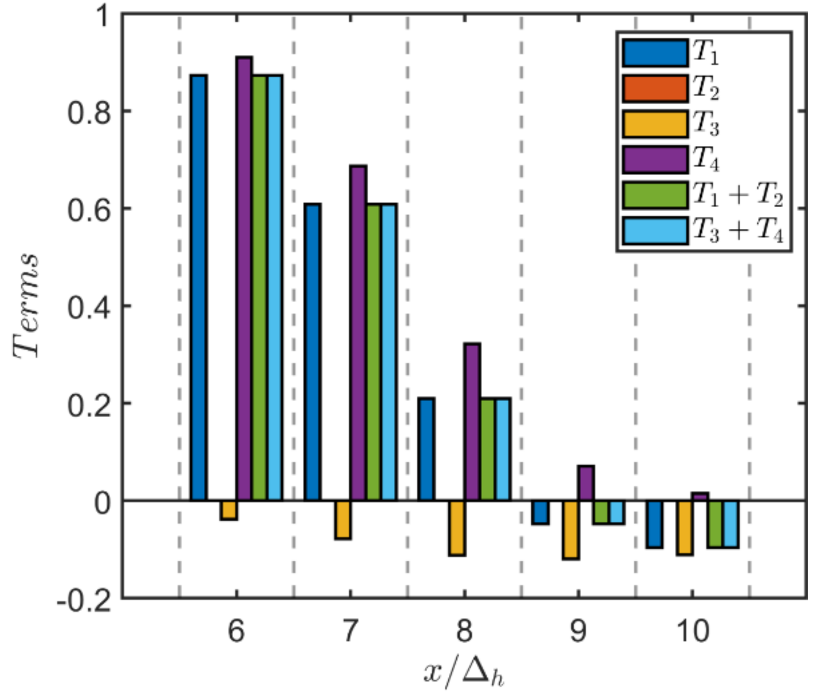
$$\underbrace{\frac{\partial}{\partial x}\int_0^{\delta_T}\frac{\bar{\rho}\tilde{u}(\tilde{h}-\tilde{h}_\infty)}{\rho_0 u_{\tau,NR}(h_{ad}-h_0)}dy}_{T_1}+\underbrace{\frac{1}{(h_{ad}-h_0)}\frac{d\tilde{h}_\infty}{dx}\int_0^{\delta_T}\frac{\bar{\rho}\tilde{u}}{\rho_0 u_{\tau,NR}}dy}_{T_2}=\underbrace{\frac{\bar{q}_w}{\rho_0 u_{\tau,NR}(h_{ad}-h_0)}}_{T_3}+\underbrace{\frac{1}{\rho_0 u_{\tau,NR}}\int_0^{\delta_T}\overline{\dot{\omega}_c}dy}_{T_4}$$

Advection Term

Freestream temperature variation term

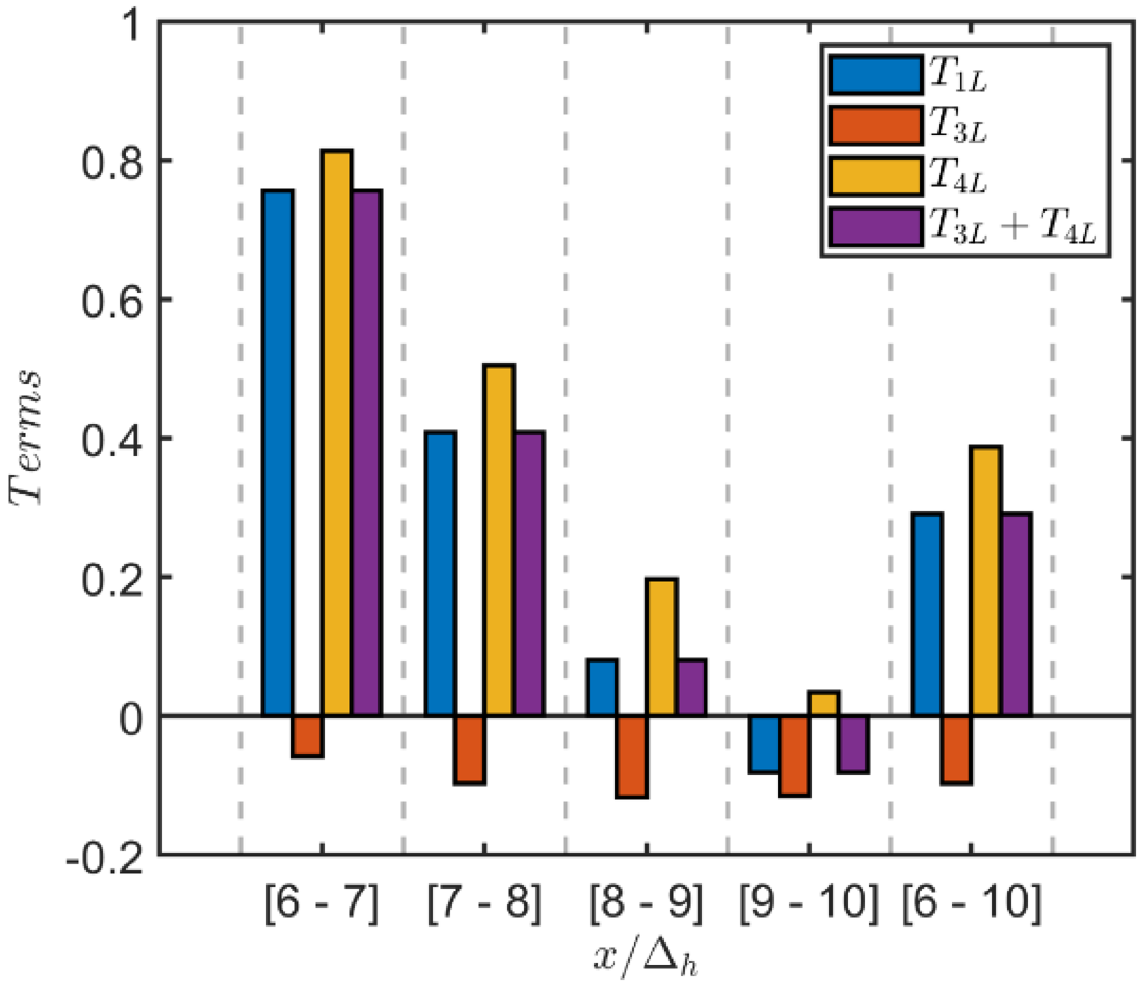
Wall heat flux term

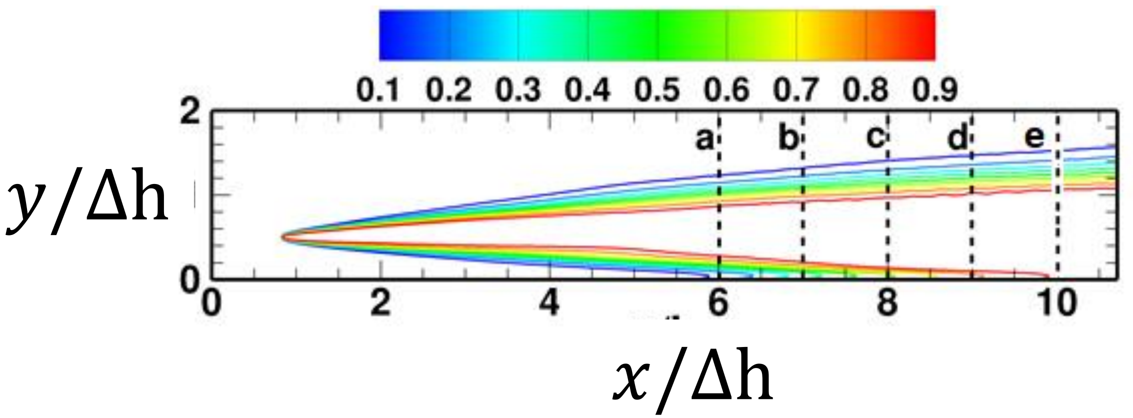
Mean reaction rate term



Variations of $T_1, T_2, T_3, T_4, T_1+T_2$ and T_3+T_4 at $x/\Delta_h = 6.0, 7.0, 8.0, 9.0$ and 10.0.

Variation of terms in the streamwise direction



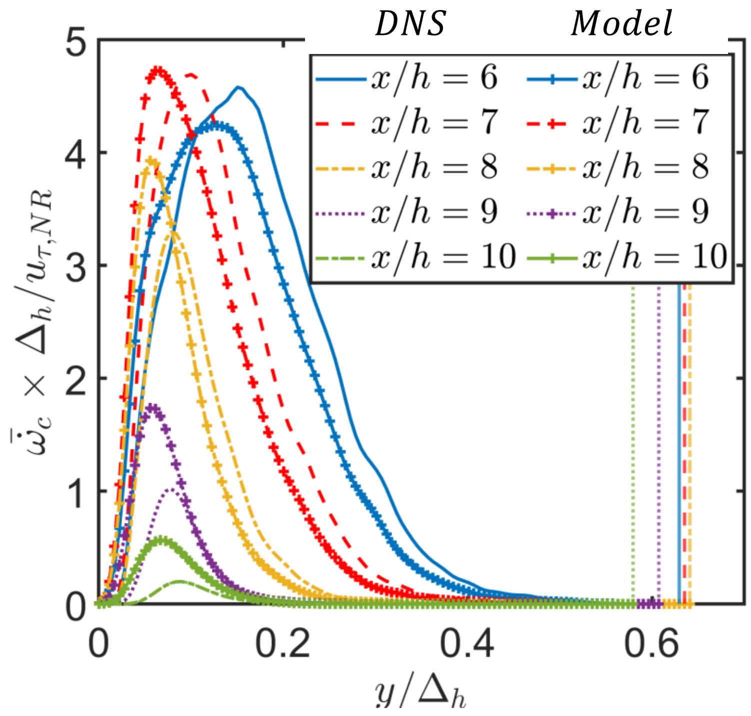


Favre Averaged Progress variable

- Location a = $x/\Delta h = 6$
- Location b = $x/\Delta h = 7$
- Location c = $x/\Delta h = 8$
- Location d = $x/\Delta h = 9$
- Location e = $x/\Delta h = 10$

Variations of T_{1L}, $T_{3L} = (-Nu_L/\{Re_\tau Pr\})$, $T_{4L} = A_{proj}S_T/[A_{seg}u_{\tau,NR}]$ and $T_{3L} + T_{4L} = -Nu_L/\{Re_\tau Pr\}$+$A_{proj}S_T/[A_{seg}u_{\tau,NR}]$ for $7.0 \geq x/\Delta_h \geq 6.0$, $8.0 \geq x/\Delta_h \geq 7.0$, $9.0 \geq x/\Delta_h \geq 8.0$, $10.0 \geq x/\Delta_h \geq 9.0$ along with the corresponding variations for $10.0 \geq x/\Delta_h \geq 6.0$.

Mean Reaction Rate



$\overline{\dot{\omega}_c} = I_0 \rho_0 S_L \Sigma_{gen}$ **(lines with symbols)**

where $I_0 = 0.5\left[\mathrm{erf}\left(y/\delta_z - Pe_Q\right) + 1\right]$[1,2]

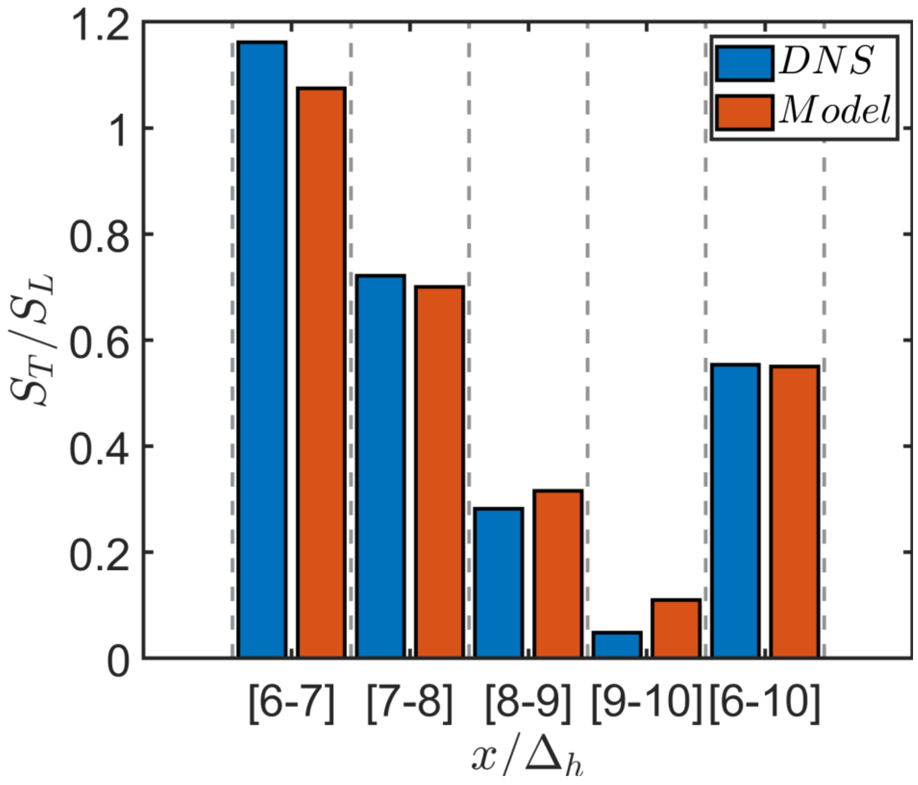
where $\delta_z = \alpha_{T0}/S_L$ is the Zel'dovich flame thickness,

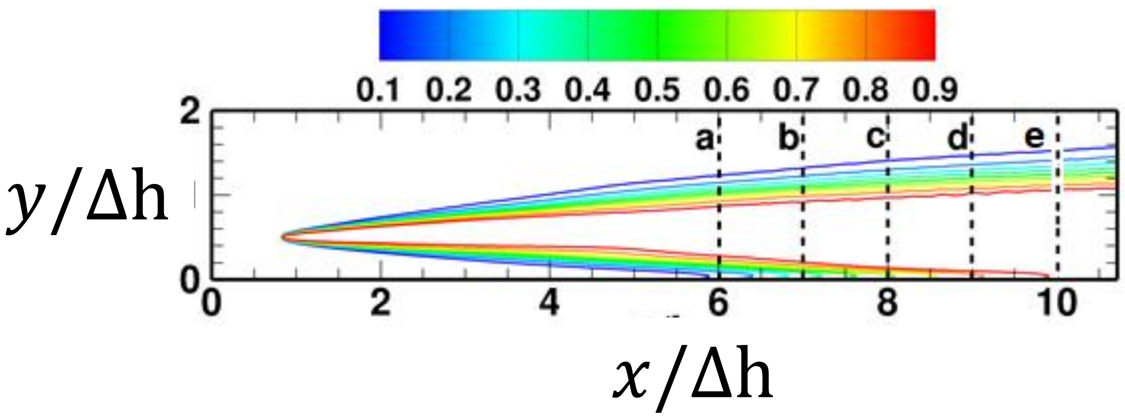
$Pe_Q = \delta_Q/\delta_z$ is the wall Peclet number for the laminar head-on quenching configuration (=2.19 for the present thermochemistry)

Variations of $\overline{\dot{\omega}_c} \times \Delta_h/\rho_0 u_{\tau,NR}$ (lines) with y/Δ_h for the bottom wall along with the predictions of $I_0 \rho_0 S_L \Sigma_{gen} \times \Delta_h/\rho_0 u_{\tau,NR}$ (lines with symbols) where $I_0 = 0.5[\mathrm{erf}\left(y/\delta_z - Pe_Q\right) + 1]$ for $x/\Delta_h =$ 6.0, 7.0, 8.0, 9.0 and 10.0. Vertical lines represents the cut-off limits of the thermal boundary layer thickness δ_t.

[1]J. Sellmann, J. Lai, A. M. Kempf, N. Chakraborty, Flame surface density based modelling of head-on quenching of turbulent premixed flames, Proc. Combust. Inst. 36 (2017).
[2]U. Ahmed, N. Chakraborty, M. Klein, Assessment of bray moss libby formulation for premixed flame-wall interaction within turbulent boundary layers: Influence of flow configuration, Combust. Flame 233 (2021).

Comparison of model and DNS prediction of turbulent burning velocity





Favre Averaged Progress variable

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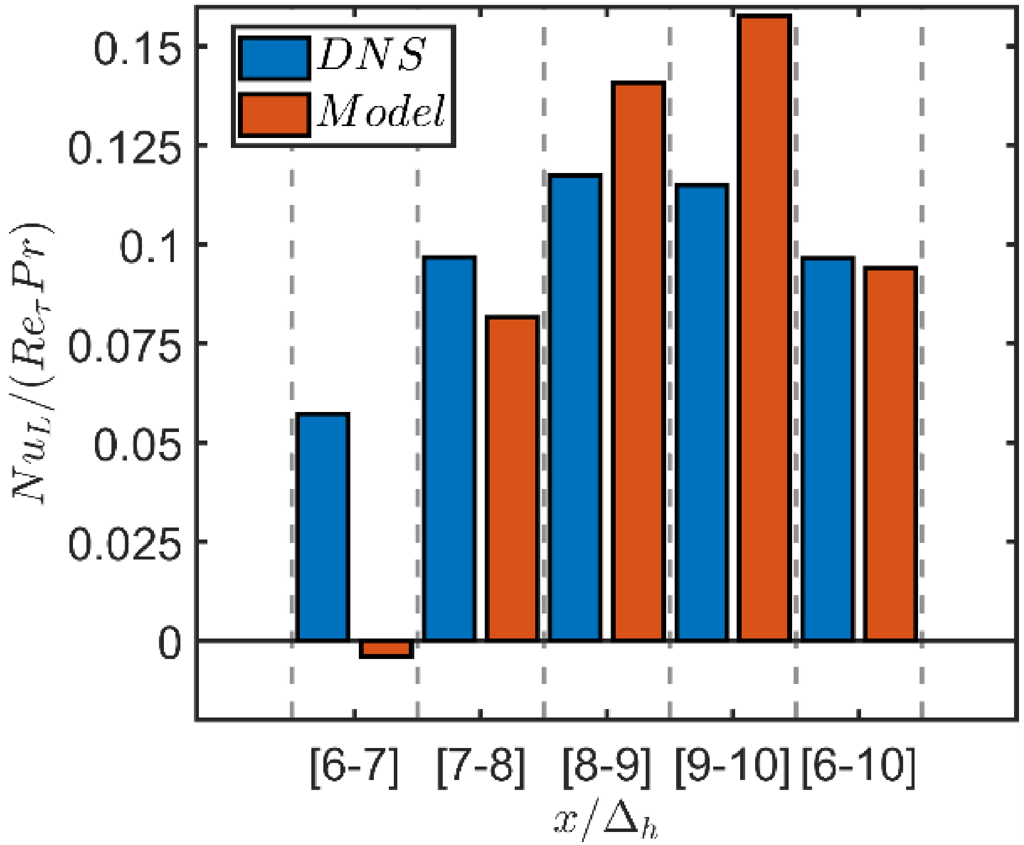
Predictions of $S_T^{model} = \left(\rho_0 A_{proj}\right)^{-1} \int_{L_1}^{L_2} \int_0^{\delta_T} I_0 \rho_0 S_L \Sigma_{gen} W dx dy$ along with $S_T = \left(\rho_0 A_{proj}\right)^{-1} \int_{L_1}^{L_2} \int_0^{\delta_T} \overline{\dot{\omega}_c} W dx dy$ extracted from DNS data for $7.0 \geq x/\Delta_h \geq 6.0$, $8.0 \geq x/\Delta_h \geq 7.0$, $9.0 \geq x/\Delta_h \geq 8.0$ $10.0 \geq x/\Delta_h \geq 9.0$ and also for $10.0 \geq x/\Delta_h \geq 6.0$.

Comparison of model and DNS prediction of Stanton number



$$T_{1L} + T_{2L} = -\left[\frac{Nu_L}{\{Re_\tau Pr\}}\right]^{\boldsymbol{Model}} + A_{proj} S_T^{model} / [A_{seg} u_{\tau,NR}]$$

Predictions of $Nu_L/\{Re_\tau Pr\}$ by using S_T^{model} in eq. 4 compared to $Nu_L/\{Re_\tau Pr\}$ extracted from DNS data.

Summary

- A new integral form of the energy conservation equation has been derived for premixed FWI within TBLs under low Mach number conditions

- It has been found that the wall heat flux and the heat release rate remain the leading order contributors to the integral form of the energy conservation equation and their net contribution is balanced by the contribution arising from the advection process

- The magnitudes of the wall heat flux increase and heat release rate contributions decrease with increasing distance from the leading edge of the thermal boundary layer due to the thickening of the boundary layer and due to the progress of the flame quenching

- The integral form of the energy conservation equation has been utilised to demonstrate that the Nusselt number (or Stanton number) for wall heat transfer is closely related to the turbulent burning velocity within TBLs

- A methodology based on the FSD based reaction rate closure revised to account for near-wall behaviour has been shown to reasonably capture the behaviour of the turbulent burning velocity within the TBL when a sufficiently large span of distance is considered

- This suggests that the measurements of mean velocity, temperature and wall heat flux can be utilised to estimate the turbulent burning velocity within TBLs

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