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Large Eddy Simulation of period-2 thermoacoustic instabilities in the PRECCINSTA burner using flamelets

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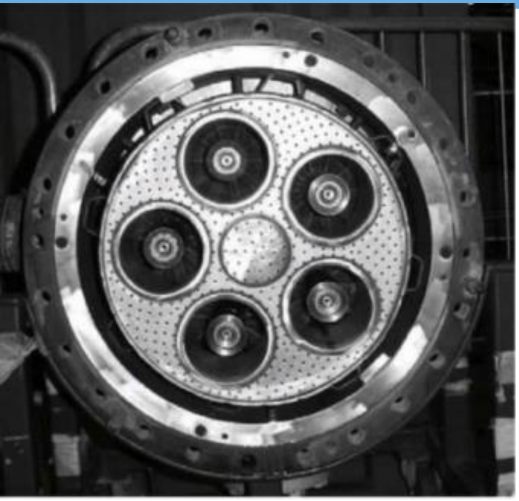
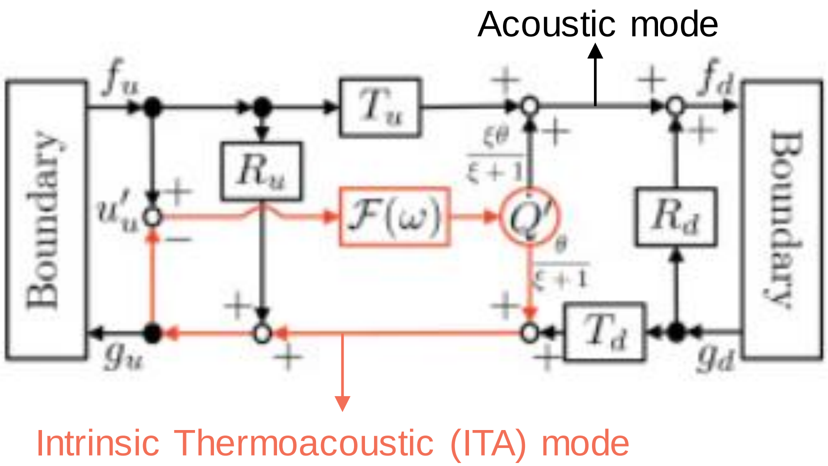
Outline

- Motivation
- Background and Aim
- Description of case
- Numerical modelling
- Results
- Analysis
- Conclusion

Motivation



[1]



[2]

- Combustion instabilities have detrimental effects on combustor life.
- Different modes can occur at multiple frequencies and is not desired from a design perspective.
- Multiple modes may show interplay and coalesce.
- Infinite sensitivities may be encountered at exceptional points in the eigenspace – both a boon and a bane!
- LES of cases close to exceptional points are yet to be performed or identified.

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1. Lieuwen, T. C., & Yang, V. (Eds.). (2005). *Combustion instabilities in gas turbine engines: operational experience, fundamental mechanisms, and modeling*. American Institute of Aeronautics and Astronautics.
2. Ghani, A., Steinbacher, T., Albayrak, A., & Polifke, W. (2019). Intrinsic thermoacoustic feedback loop in turbulent spray flames. *Combustion and Flame*, *205*, 22-32.

Background and Aim

$$\nabla.(c^2\nabla\hat{p}) + \omega^2\hat{p} = -\mathrm{i}\omega(\gamma - 1)\hat{\dot{q}}$$ [3]

Nonlinear Eigenvalue Problem ← $\boldsymbol{L}(\omega;\epsilon)\widehat{\boldsymbol{p}} = \boldsymbol{0}$ $\quad \epsilon \in \mathbb{R}^M; \boldsymbol{L} \in \mathbb{C}^{N\times N}$

Non-Hermitian ($\boldsymbol{L_{ij}} \neq \boldsymbol{L^*_{ji}}$)

$\omega = \omega_r - \mathrm{i}(-\omega_i)$

Angular Frequency (ω_r)

Growth rate ($-\omega_i$)

[4]

a

Distinct

Repeated

$a = g$

$a > g$ → Defective eigenvalue

a : Algebraic Multiplicity of roots of dispersion relation

g : Geometric multiplicity – Number of linearly independent eigenvectors.

E.g. $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ has a double eigenvalue ($a = 2$), but only one distinct eigenvector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ [5]

Defective eigenvalues that are branch-point singularities in parameter space are called Exceptional Points (EP).

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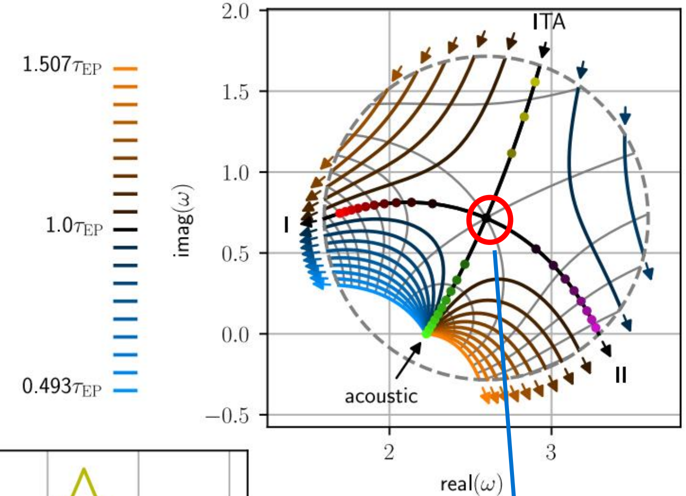
3. Crighton, D. G., Dowling, A. P., Williams, J. E., Heckl, M., & Leppington, F. G. (1992). Thermoacoustic sources and instabilities. In *Modern methods in analytical acoustics* (pp. 378-405). Springer, London.
4. Leung, A. Y. T. (1990). Perturbed general eigensolutions. *Communications in applied numerical methods*, *6*(5), 401-409.
5. Mensah, G. A., Magri, L., Silva, C. F., Buschmann, P. E., & Moeck, J. P. (2018). Exceptional points in the thermoacoustic spectrum. *Journal of Sound and Vibration*, *433*, 124-128.

Background and Aim

Taylor series expansion is not possible around defective eigenvalues, instead fractional power series (Puiseux series) is used. [4]

$$\omega = \omega_{\mathrm{EP}} + \omega_1\left(\epsilon_i - \epsilon_{i,\mathrm{EP}}\right)^{1/a} + \omega_2\left(\epsilon_i - \epsilon_{i,\mathrm{EP}}\right)^{2/a} + \ldots$$



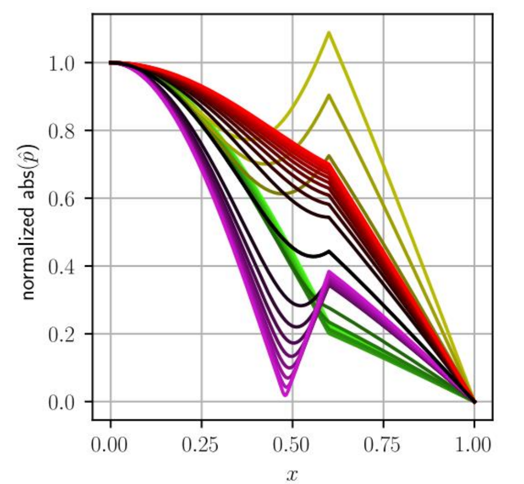
EP

[5]

Physical implications:

- $\left.\frac{\partial\omega}{\partial\epsilon_i}\right|_{\epsilon_{i,\mathrm{EP}}} \rightarrow \infty$. Infinite sensitivity to change in parameters. Capturing modes close to EP may be challenging due to such a high sensitivity.

- Eigenvalue and eigenvectors coalesce. ITA and acoustic modes could coalesce and mode shapes can have characteristics of both ITA and acoustic mode.



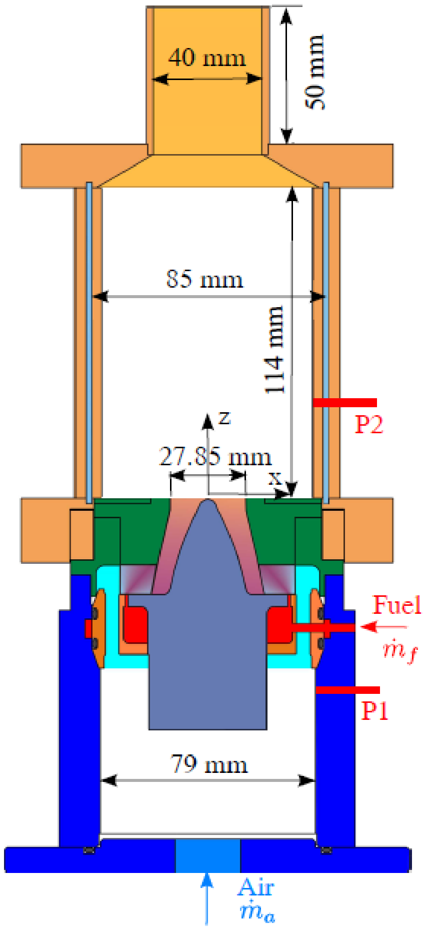
Can LES based on flamelets capture a mode close to an EP?

Can LES capture period-2 oscillations from 2 modes?

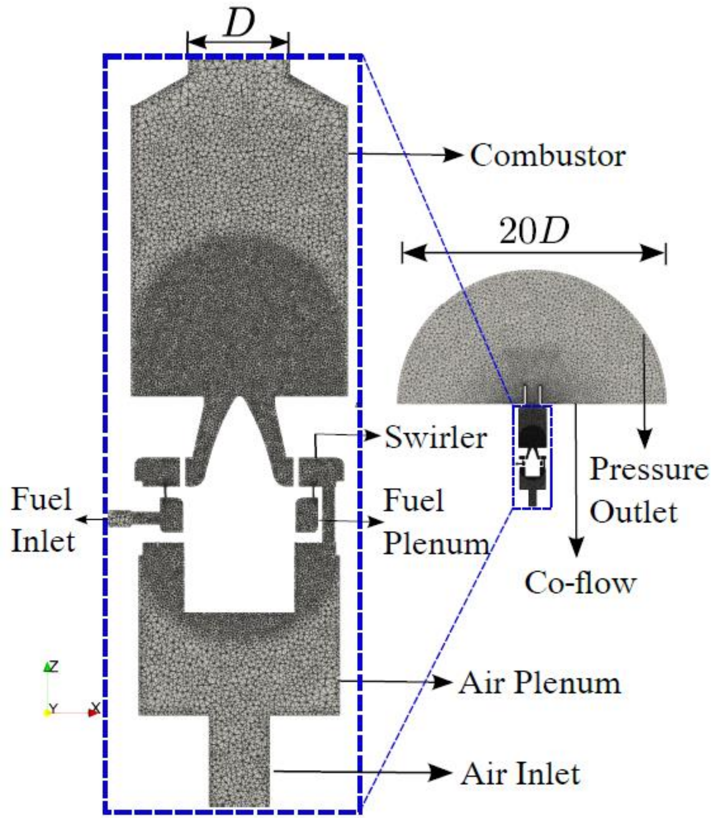
4. Leung, A. Y. T. (1990). Perturbed general eigensolutions. *Communications in applied numerical methods*, *6*(5), 401-409.
5. Mensah, G. A., Magri, L., Silva, C. F., Buschmann, P. E., & Moeck, J. P. (2018). Exceptional points in the thermoacoustic spectrum. *Journal of Sound and Vibration*, *433*, 124-128

Description of case and mesh

[6]



Case	ϕ_{glob}	$\dot{m}_a(\mathrm{g/min})$	$\dot{m}_f(\mathrm{g/min})$	$P_{\mathrm{therm}}(\mathrm{kW})$
TP09	0.9	574	30	25



- 4.5 million unstructured tetrahedral cells.
- Hemispherical domain to mimic external atmosphere.
- $\Delta x = 1\mathrm{mm}$ in the combustion zone and swirler region.
- Co-flow velocity of 0.1m/s at the base of the hemispherical domain.

6 Dem, C., Stöhr, M., Arndt, C. M., Steinberg, A. M., & Meier, W. (2015). Experimental study of turbulence-chemistry interactions in perfectly and partially premixed confined swirl flames. *Zeitschrift für Physikalische Chemie*, *229*(4), 569-595.

Numerical modelling

Governing equations

- Favre-filtered mass and momentum equations are solved using a dynamic Smagorinsky turbulence model.

- Mixture fraction, progress variable, their respective variances and thermo-chemical enthalpy are transported using:

$$\overline{\rho}\frac{D\tilde{\varphi}}{Dt} = \nabla.\left[\left(\overline{\mu} + \frac{\mu_T}{Sc_T}\right)\nabla\tilde{\varphi}\right] + \overline{S_{\varphi}^{+}} - \overline{S_{\varphi}^{-}}$$

where $\tilde{\varphi} = \left\{\tilde{\xi},\ \sigma_{\xi,\mathrm{sgs}}^2, \tilde{c}, \sigma_{c,\mathrm{sgs}}^2, \tilde{h}\right\}$

$$\overline{S_{\varphi}^{+}} = \left\{0, 2\frac{\mu_T}{Sc_T}\left|\nabla\tilde{\xi}\right|^2, \overline{\dot{\omega}^*}, 2\frac{\mu_T}{Sc_T}|\nabla\tilde{c}|^2 + 2\left(\overline{c\ \dot{\omega}_c^*} - \tilde{c}\overline{\dot{\omega}_c^*}\right), \frac{D\overline{p}}{Dt}\right\}$$

$$\overline{S_{\varphi}^{-}} = \left\{0, 2\overline{\rho}\tilde{\chi}_{\xi,\mathrm{sgs}}, 0, 2\overline{\rho}\tilde{\chi}_{c,\mathrm{sgs}}, 0\right\}$$

$$\overline{\dot{\omega}_c^*} = \overline{\dot{\omega}_c} + \overline{\dot{\omega}_{\mathrm{np}}} = \overline{\rho}\int_0^1\int_0^1 \frac{\dot{\omega}_c(\eta,\zeta)}{\rho(\eta,\zeta)}\tilde{P}(\eta,\zeta)d\eta d\zeta + \tilde{c}\left(\frac{\overline{\mu}}{Sc}\left|\nabla\tilde{\xi}\right|^2 + \overline{\rho}\tilde{\chi}_{\xi,\mathrm{sgs}}\right) \times \int_0^1 \frac{1}{\psi^{\mathrm{Eq}}}\frac{d^2\psi^{\mathrm{Eq}}}{d\eta^2}\tilde{P}_{\beta}(\eta)d\eta$$

Code(s) used: OpenFOAM v7 / FlaRe

Time usage on ARCHER (approx. CUs): 3000 CUs

- Flamelet model (FlaRe) maps the thermochemical states into mixture-fraction progress variable ($\xi - c$) space.

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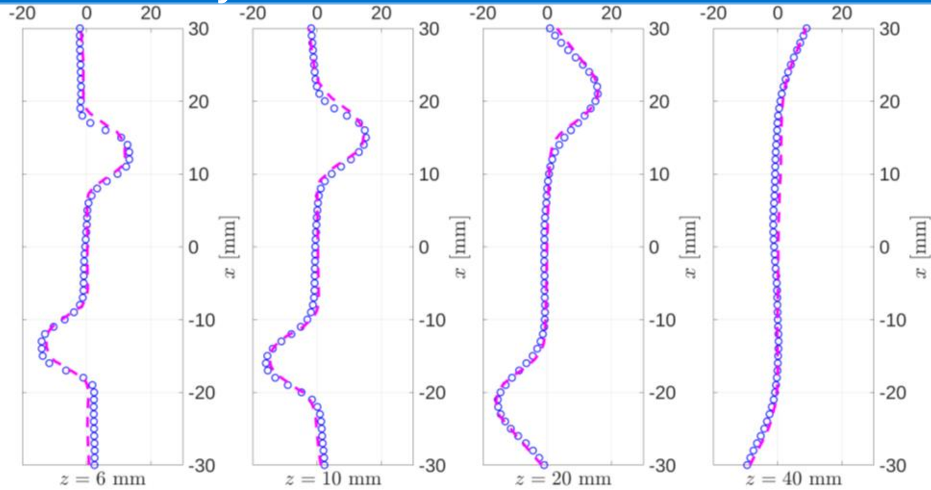
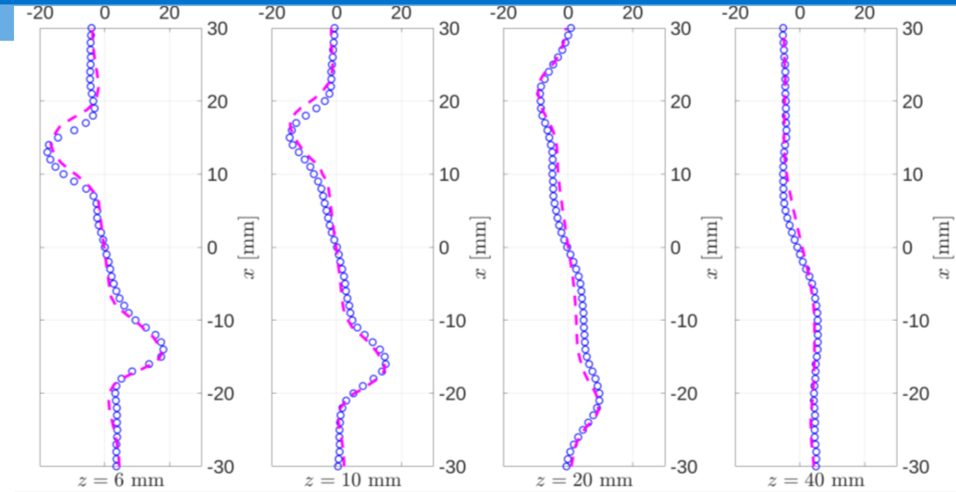
7. Chen, Zhi, Shaohong Ruan, and Nedunchezhian Swaminathan. Large eddy simulation of flame edge evolution in a spark-ignited methane–air jet. *Proceedings of the Combustion Institute* 36.2 (2017): 1645-1652.

Results

Velocity statistics - Mean



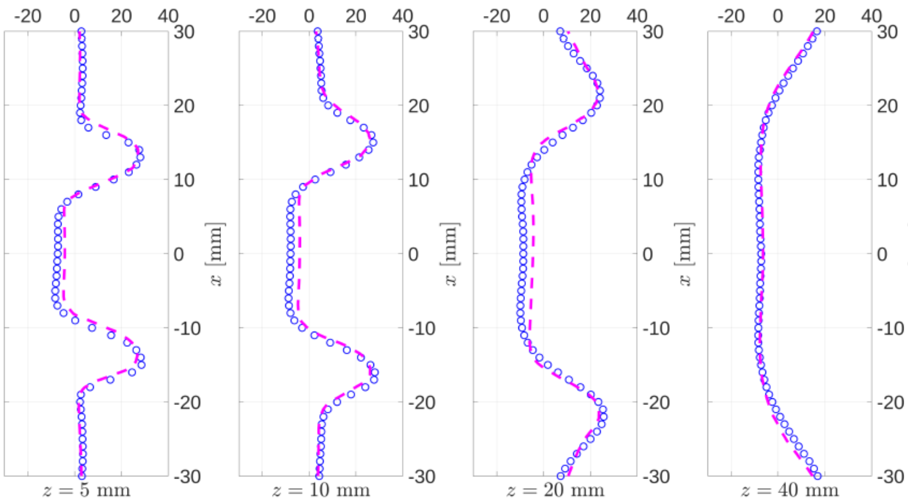
$\overline{U}_x(\mathrm{ms}^{-1})$

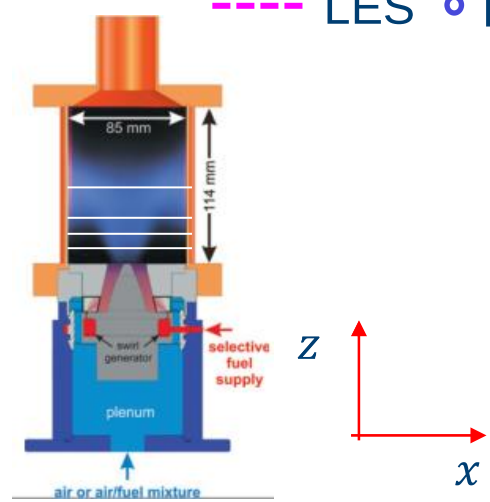


$\overline{U}_y(\mathrm{ms}^{-1})$

LES EXP

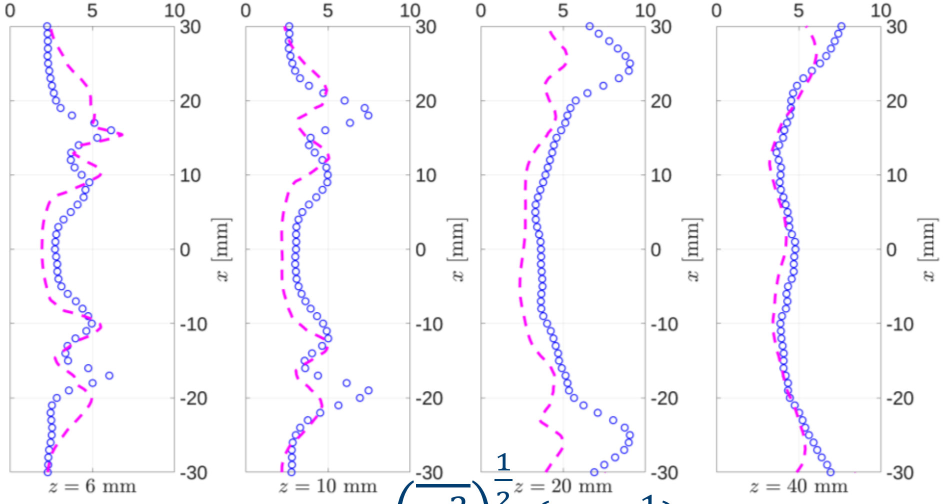
$\overline{U}_z(\mathrm{ms}^{-1})$



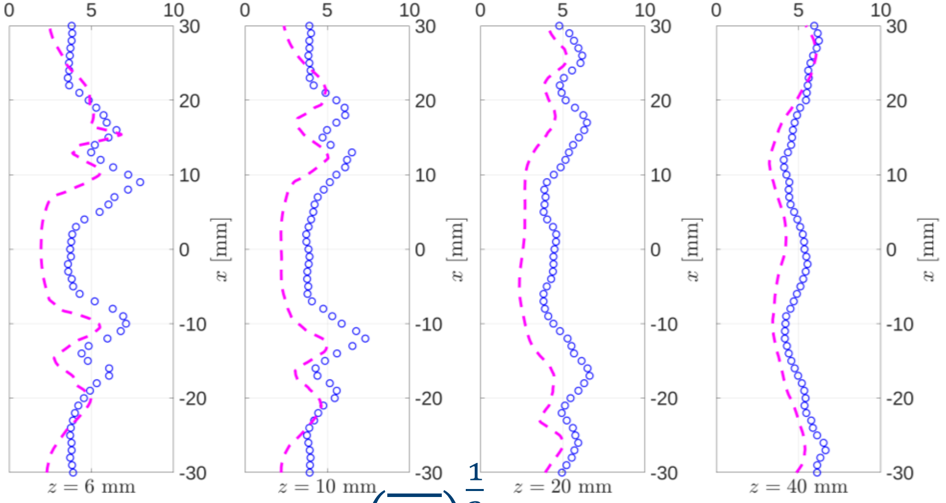


Results

Velocity statistics - RMS

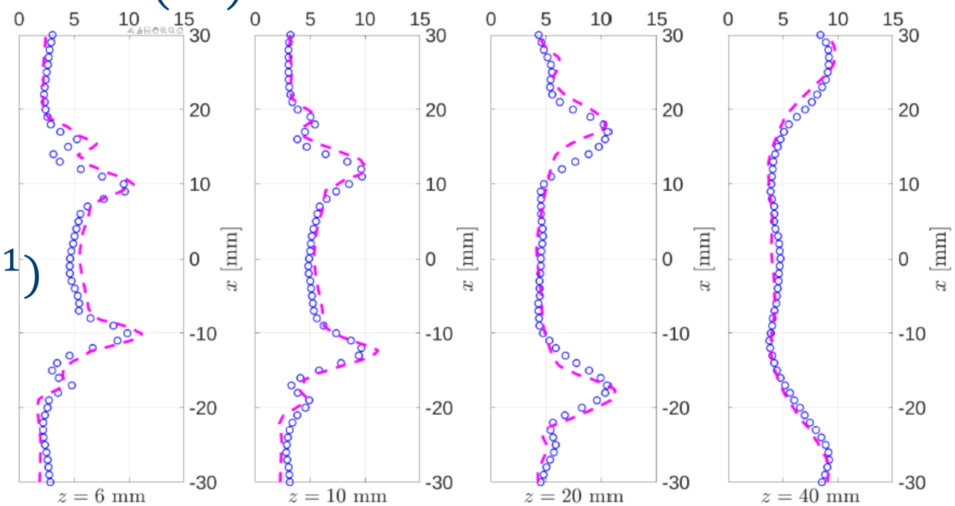


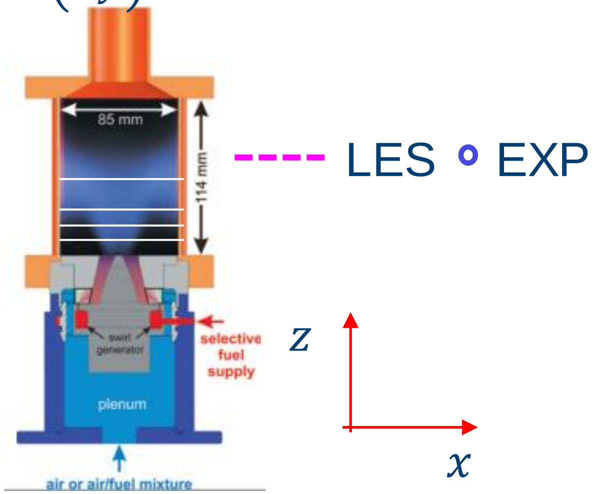
$\left(\overline{\sigma_x^2}\right)^{\frac{1}{2}}$ (ms^{-1})



$\left(\overline{\sigma_y^2}\right)^{\frac{1}{2}}$ (ms^{-1})

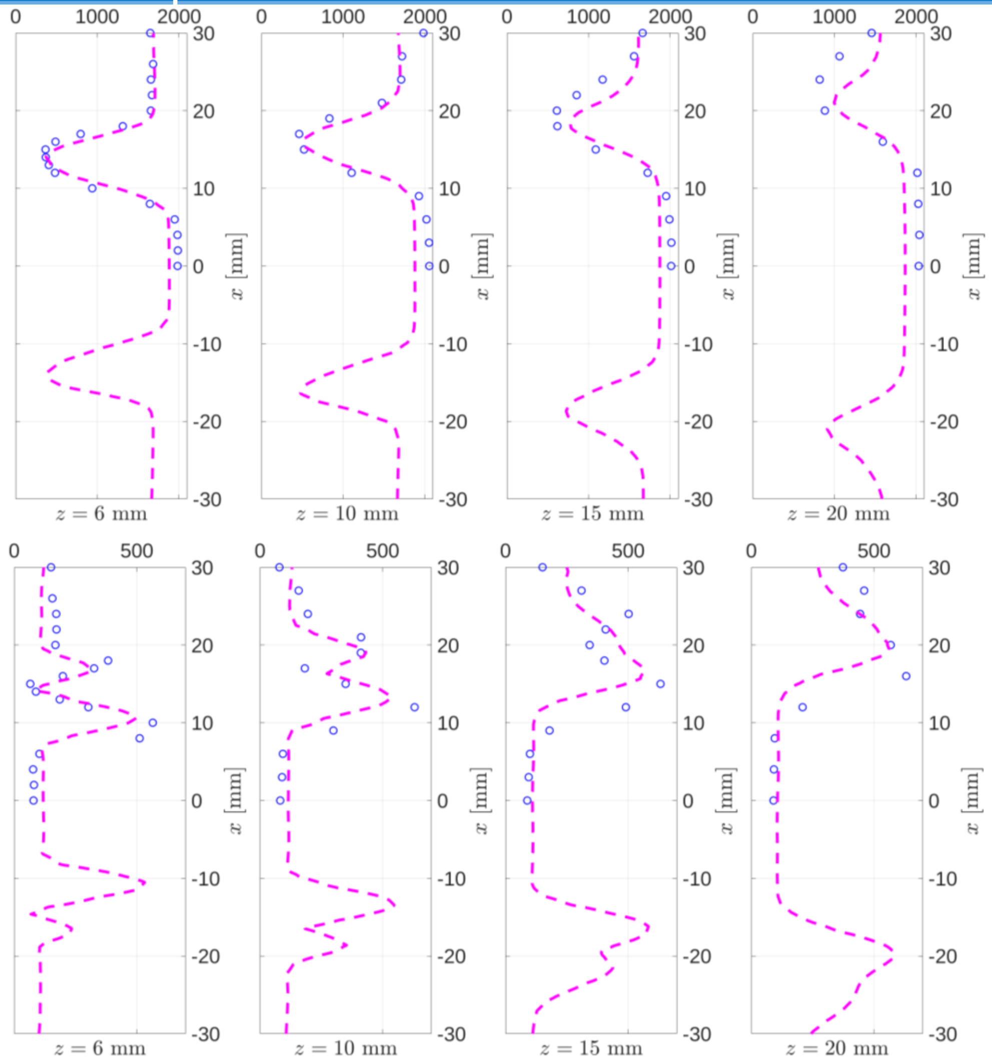
$\left(\overline{\sigma_z^2}\right)^{\frac{1}{2}}$ (ms^{-1})





Results

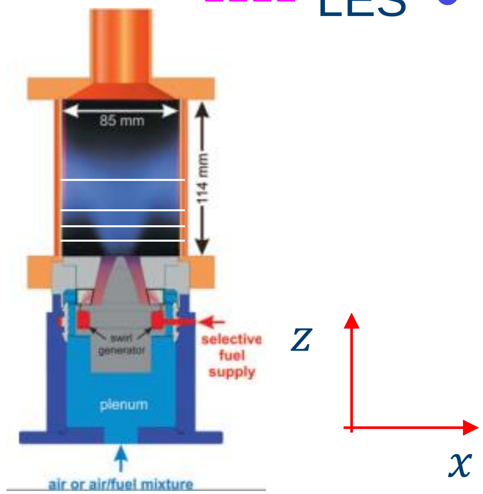
Temperature statistics



$\overline{T}$ [K]

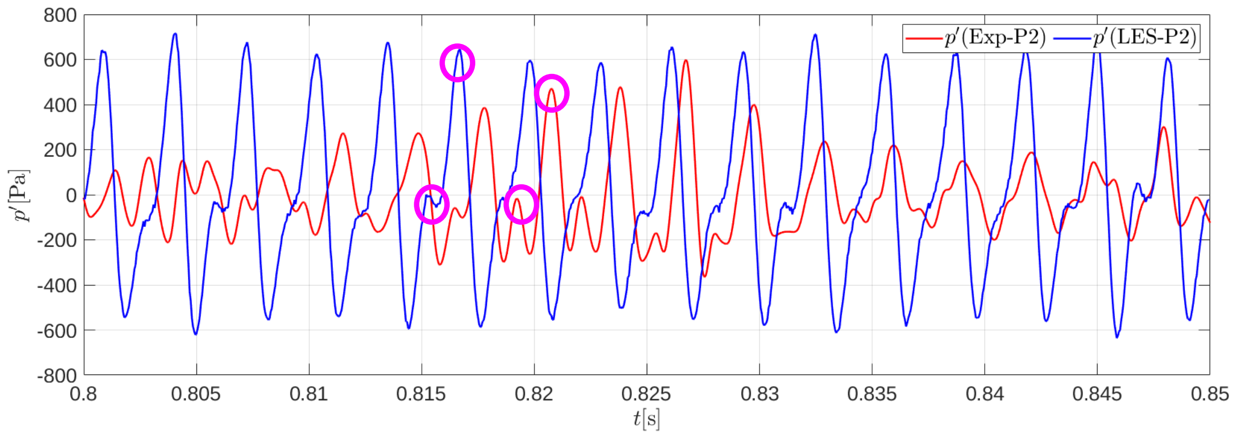
$\sqrt{\overline{\sigma_T^2}}$ [K]

LES EXP

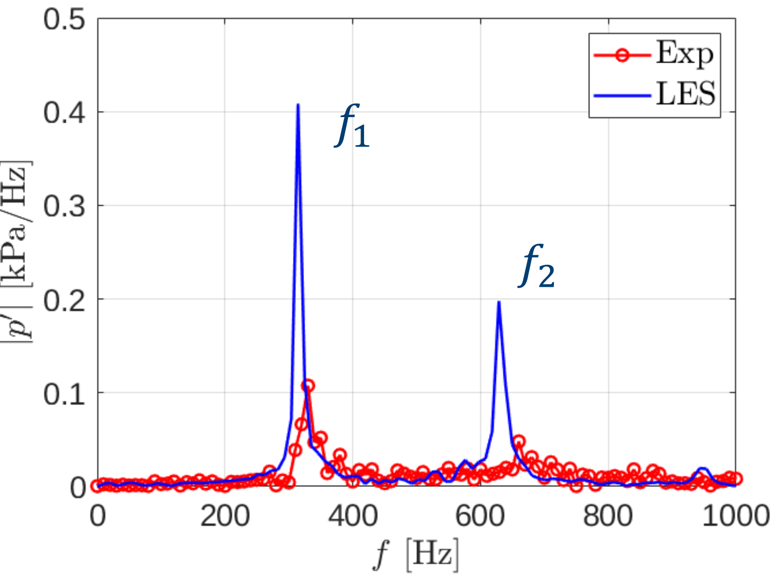


Results

Pressure time series



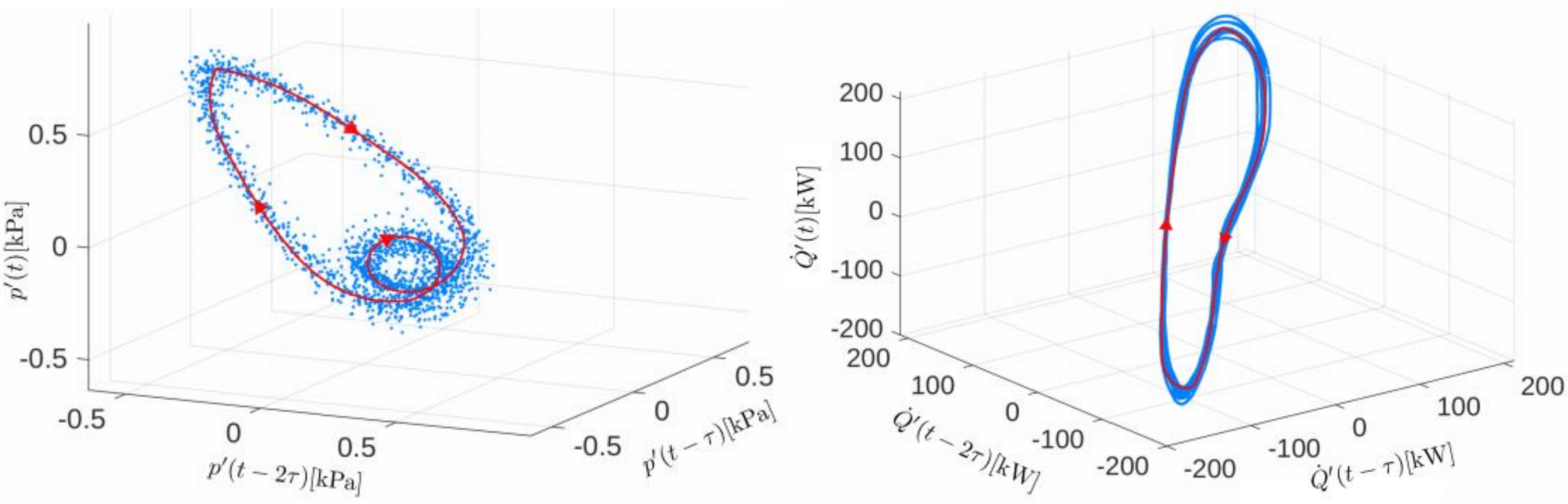
$f_1 \approx$ 314 Hz; $f_2 \approx$ 628 Hz



- Two dominant frequencies observed in both LES and EXP.
- Pressure amplitude is lower in EXP due to damping from loosely fitted quartz windows and some heat loss.
- Pressure amplitude is also time varying in EXP.

Analysis

Three-dimensional phase space



$$I(\tau) = \sum_{f(t_i),f(t_i+\tau)} P(f(t_i), f(t_i+\tau)) \log \left[\frac{P(f(t_i), f(t_i+\tau))}{P\big(f(t_i)\big).P(f(t_i+\tau))} \right] \quad [8]$$

- τ is the location of the first local minima of Average Mutual Information ($I(\tau)$).
- 3D phase space of pressure shows a double-loop attractor whereas HRR shows a single-loop attractor. HRR does not couple with the second mode.

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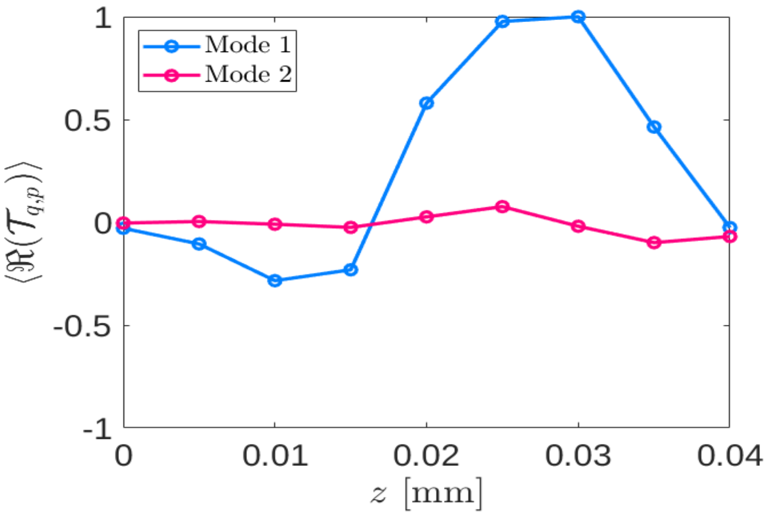
8. Fraser, A. M., & Swinney, H. L. (1986). Independent coordinates for strange attractors from mutual information. *Physical review A*, *33*(2), 1134.

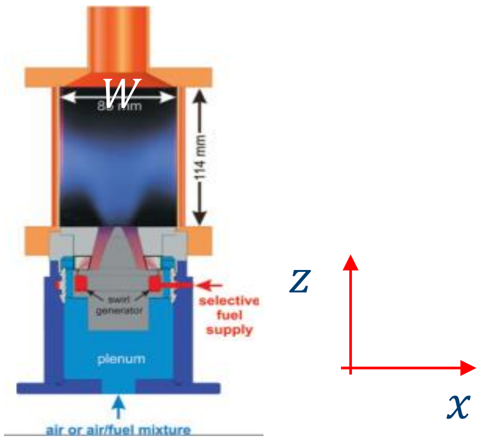
Analysis

Rayleigh index



$$RI(f_i) = \langle \Re(\mathcal{T}_{q,p}(f_i)) \rangle = \frac{1}{W} \int \Re\left(\widehat{p'}(f_i).\,\widehat{\dot{q}'}(f_i)\right) dx$$

[9]

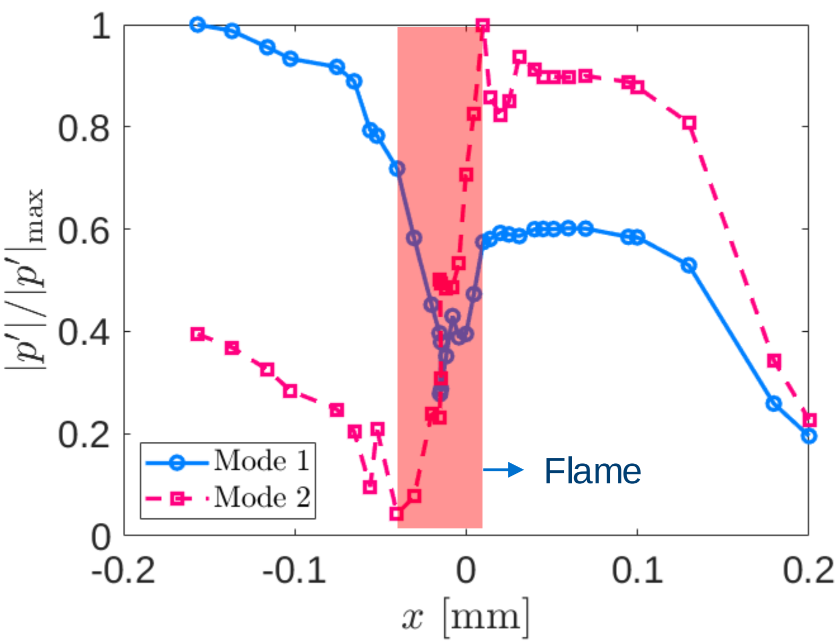


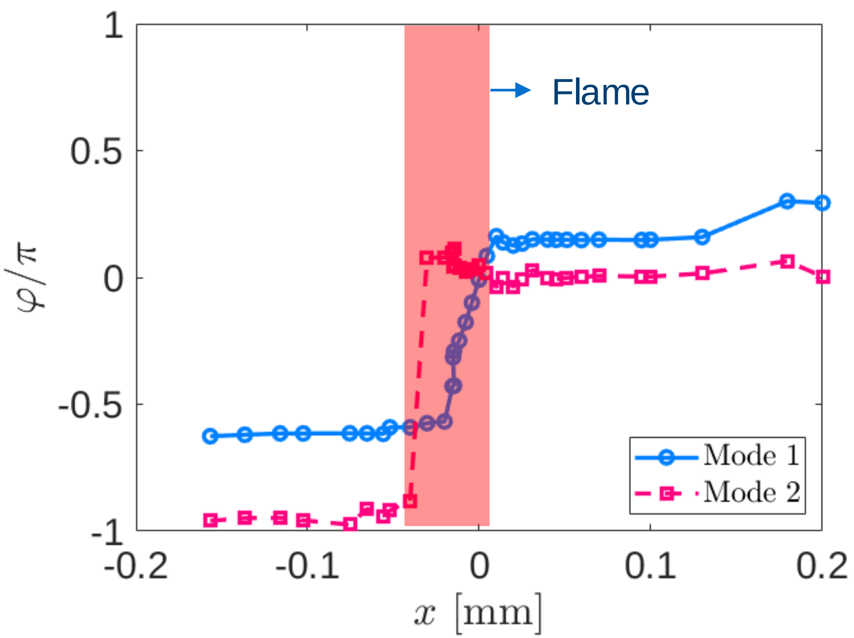
- Rayleigh Index shows the strength of the coupling between pressure and heat release fluctuations.
- Mode 1 shows a overall net positive coupling whereas mode 2 shows negligible coupling.
- This explains the single-loop attractor of heat release rate in the presence of a double-loop attractor of pressure.

9. Rayleigh, L. (1878). The explanation of certain acoustical phenomena. *Roy. Inst. Proc.*, *8*, 536-542.

Analysis

Mode shapes





- Mode 2 has the characteristic shape of ¼ wave mode.
- Mode 1 shows a peculiar shape that has the characteristics of both ITA (Intrinsic Thermoacoustic Mode) and a Helmholtz mode.
- Therefore, this mode has to be closed to the veering region caused by an exceptional point (EP).

Conclusions

- LES using FlaRe model shows good agreement with PIV and Raman measurements for velocity and Temperature statistics, respectively.

- LES captures period-2 oscillations observed in the experiment albeit with an overprediction in amplitudes due to lack of damping in LES.

- Phase space reconstruction of pressure and HRR reveal double-loop and single-loop attractors. Rayleigh index shows strong coupling of pressure and HRR only at f_1.

- Mode 1 resembles a thermoacoustic mode close to an EP and mode 2 resembles a quarter-wave mode.

- LES-FlaRe is able to successfully capture period-2 behaviour and a thermoacoustic mode close to an EP.

Thank you for listening. Any questions?

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